No Reliance on Guidance: Counter-Signaling in Management Forecasts

Cyrus Aghamolla* Carlos Corona† Ronghuo Zheng‡

August 13, 2019

Abstract

This study presents and provides an explanation for a novel stylized fact: both high-performing public companies as well as troubled companies withhold issuing guidance. We assume that the manager's ability affects the level of earnings and the accuracy of the guidance, but issuing a forecast is *costless* for all manager types. Managers are thus able to signal their ability through accuracy in their forecasts. While high ability managers would seem to benefit the most from issuing guidance, in equilibrium we find that both high and low ability managers *withhold* guidance, while intermediate ability managers issue forecasts. This occurs since high ability managers do not need to rely on guidance in order to convey their ability to the market, while intermediate managers must forecast to separate from low ability managers. Hence, we find that high ability managers *counter-signal* in equilibrium by withholding guidance, which does not result in a subsequent "punishment" by the market. Additionally, the results offer new empirical predictions.

Keywords: Counter-signaling, disclosure, earnings guidance, management forecasts. *JEL classification*: C72, D82, D83, G14.

*University of Minnesota. E-mail: caghamol@umn.edu.
†Carnegie Mellon University. E-mail: ccorona@andrew.cmu.edu.
‡University of Texas at Austin. Email: ronghuo.zheng@mccombs.utexas.edu.

"Although we may discuss long term trends in our business, we do not plan to give earnings guidance in the traditional sense. ... We would prefer not to be asked to make such predictions, and if asked we will respectfully decline."

——Larry Page and Sergey Brin, Google co-founders, 2004.

1 Introduction

The practice of issuing earnings guidance has become prevalent among U.S. public firms, with 76% of firms issuing some kind of guidance. However, a curious anomaly has arisen in recent years: several well-known and largely successful public companies have either not adopted the practice after their IPO (such as Google), or discontinued the practice entirely, such as Costco, Ford, UPS, Coca-Cola, AT&T, and Berkshire Hathaway.[1] Meanwhile, a sizable empirical literature documents that firms that discontinue issuing guidance, *on average*, face a severely negative reaction from the market (e.g. Cheng, Subramanyam, and Zhang, 2005; Houston, Lev, and Tucker, 2010; Brochet, Faurel, and McVay, 2011; Chen, Matsumoto, and Rajgopal, 2011). The general conclusion from these studies is that firms stop issuing guidance as they expect negative shocks or downturns in their operations, thus aiming to limit the future voluntary release of bad news. A natural question thus arises: Why do top-performing firms choose to withhold issuing guidance, even if their future performance is not expected to decline? More generally, what are the economic forces that dictate the decision to issue guidance and the subsequent market pricing? The purpose of this paper is to shed light on these questions.

The anecdotal evidence of highly successful firms withholding guidance is supported by a descriptive examination of the data. We identify and present a new stylized fact that is consistent with the aforementioned anecdotal evidence. The graph in Figure 1 represents the proportion of firms that issue guidance at different levels of contemporaneous profitability. Firms provide guidance with the highest likelihood at intermediate levels of profitability, while both low-performing and high-performing firms provide guidance with lower frequencies. Although the extant literature has documented that poorly performing

[1] See "Giving up on Guidance," *Bloomberg Businessweek*, May 14, 2007.

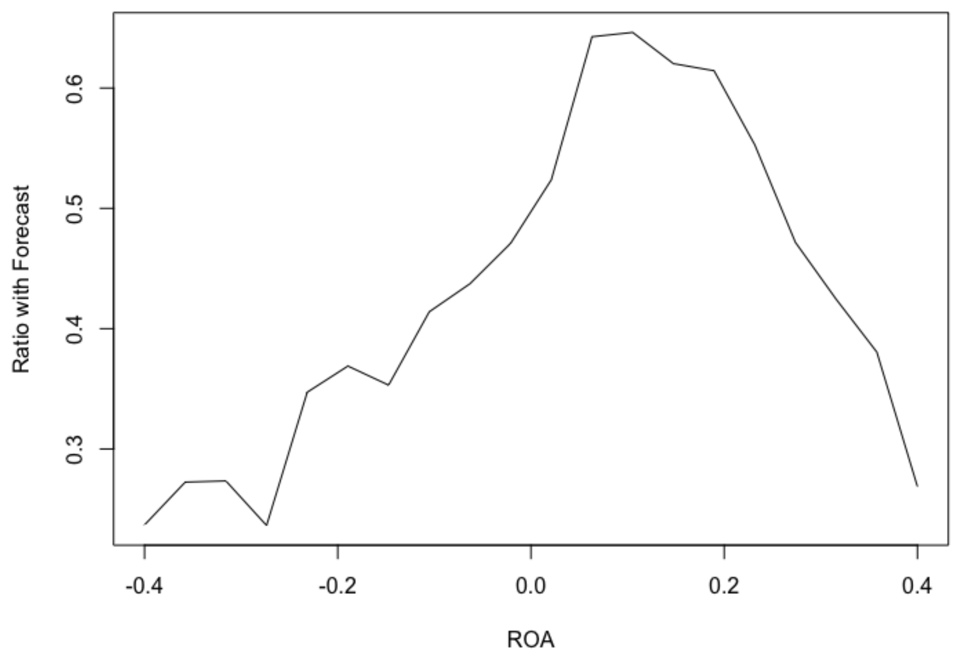

Figure 1: We classify all firm-years according to their return on assets (x-axis; ROA) into 20 bins (low to high). For each bin, we divide the corresponding number of forecasting firm-years by the total number of firm-years (y-axis; ratio with forecast). This means that the earnings guidance decision of a firm and that same firm's ROA for that year is a single firm-year data point. We collect the firm-years from 1995 to 2010 using the Company Issued Guidance (CIG)/IBES guidance database.

firms tend to reduce or discontinue guidance, the right-hand side of the graph in Figure 1 shows a new pattern. In this paper, we aim not to examine such a pattern empirically, but to show analytically how the nature of earnings guidance decisions renders this pattern.

Our setting is one where a firm manager, if endowed with discretion over the forecasting policy, must decide whether or not to issue earnings guidance.[2] The manager aims to maximize the price of the firm. The true value of the firm is increasing in the manager's ability, or type, which is privately known only by the manager. Before the market prices the firm, a mandatory earnings announcement is released, allowing the market to assess the accuracy of the forecast (if issued). The model has two main assumptions. The first main assumption is that the manager's ability affects the firm's earnings level, as well as the

[2]The friction that prevents unraveling is that we assume the manager selects a disclosure strategy prior to observing the earnings signal. This is further discussed in Section 2.

precision of the manager's forecast/guidance. Specifically, managers with a higher ability generate higher levels of earnings and can provide more precise forecasts of future earnings. The latter part of this assumption is motivated by recent empirical evidence, which finds that managers who have superior knowledge of their firm or who can more precisely predict the returns on their investments are also more capable in selecting profitable projects (Goodman, Neamtiu, Shroff, and White, 2013).[3] The large-scale survey of executives at U.S. public firms by Graham, Harvey, and Rajgopal (2005, p. 5) finds that practitioners similarly view accuracy in forecasts as indicative of competency: "If the firm had previously guided analysts to the EPS target, then missing the target can indicate that a firm is managed poorly in the sense that it cannot accurately predict its own future."

The second main assumption is that the manager may not have discretion over the guidance policy. Specifically, managers without discretion must disclose, whereas managers with discretion can set their firm's disclosure policy. This is meant to capture the notion that the guidance policy may be set by the board of directors or be so deeply entrenched in the firm's reporting practices that top management effectively cannot change the policy without broader approval. Anecdotal evidence suggests that this is not uncommon; for example, *Institutional Investor* reports that several companies have board-level committees which decide the firm's policy on earnings guidance.[4]

We find that the unique partial separating equilibrium exhibits *counter-signaling* under certain conditions. Consider the setting with three types of managers: low, medium, and high. In a counter-signaling equilibrium, among those managers endowed with discretion, only the medium type forecasts. The high type (in expectation) has the most accurate forecast as well as the highest earnings. Given that issuing guidance is *costless* for all manager types, it seems natural that the high type can best distinguish herself by issuing a precise forecast. However, we find conditions under which the opposite is true. To see this,

[3] A similar study by Baik, Farber, and Lee (2011) finds that the earnings guidance issued by higher-ability managers tends to be more precise.

[4] See "The Best CFOs in America," *Institutional Investor,* February 12, 2004. While this assumption is not central for several of our results (indeed, we show cases where the counter-signaling equilibrium uniquely exists under full discretion), it allows for more general distributions of the forecast error. This occurs because, if at least some managers have no discretion, the market always factors in the earnings announcement when updating its beliefs under any equilibrium, which we find to be a realistic equilibrium property. We discuss where this assumption plays a role more specifically in Section 4.

first note that the low type manager has an incentive to withhold guidance, as providing an imprecise forecast with a large error would make it difficult for her to be perceived as a higher type. Thus, suppose that the low type withholds guidance. Now, if the high type issues guidance and the low type withholds, the medium type would then prefer to issue guidance, as she prefers to pool with the high type rather than the low. However, if this is the case, the high ability manager would then prefer not to forecast, as she can better separate herself from the low type by relying solely on the mandatory earnings release.

In the resulting equilibrium, managers of both high and low ability choose not to issue guidance, while managers of intermediate ability choose to forecast. Our model of earnings guidance depicts an equilibrium under which high-type firms counter-signal, thus capturing the observed behavior of successful companies choosing not to forecast, despite the prevalence of earnings guidance. We thus provide a theoretical basis for the aforementioned stylized fact where intermediate-performing firms have the highest frequency of forecasts.

We first provide general theoretical results that hold for general distributions of the forecast error and of earnings which satisfy natural monotonicity assumptions. Our first main result specifies that *counter-signaling is the only partial separating equilibrium that can be supported under partial discretion* (Theorem 1). Our second general result (Theorem 2) correspondingly characterizes the set of equilibria under the assumption that all managers have discretion over the guidance policy (henceforth, full discretion).

In further analysis, we consider two common distributions for the forecast errors—the normal and uniform distributions—to examine the precise conditions under which counter-signaling occurs. The conditions are qualitatively similar under both distributions. These conditions are the following. First, the expected forecast error for low type firms must be sufficiently high relative to that of medium types. This comports with empirical evidence which finds that firms which have low performance are also poor forecasters (Goodman et al., 2013), and withhold guidance more often (Houston et al., 2010). Second, the ex ante pool (i.e., the prior probability) of low type firms must be sufficiently high. This is in line with empirical evidence, which finds an *average* price drop for non-guiding firms (Chen et al., 2011), and that firms which stop issuing guidance on average perform poorly in the time following (Houston et al., 2010). Third, expected future earnings for intermediate firms must be sufficiently high so that the medium type has an incentive to separate from the low type through providing guidance. Lastly, the manager is sufficiently likely to have discretion over the guidance decision. Overall, the conditions and the properties of counter-signaling in our setting are reasonably consistent with empirical evidence.

To see where the assumption of probabilistic discretion plays a role, consider the previous discussion with three manager types and full discretion. In the counter-signaling equilibrium, only the medium type manager issues a forecast. Consequently, the market identifies any forecasting manager as a medium type, without having to rely on the the forecasted number or the earnings realization. Because issuing guidance is costless, the low ability manager can therefore deviate by issuing a forecast, in which case the market perceives her to be an intermediate type. This is no longer true if we allow the manager to lack forecasting discretion with some probability, however small (but positive). In this case, the market always updates its beliefs using both the forecast (when issued) *and* the earnings announcement, driving the low type to pool with the high type by withhold guidance in equilibrium.

We note that our modeling assumptions do not involve monotonic cost assumptions that are typically found in signaling models.[5] Moreover, we do not invoke the conventional *single-crossing* condition.[6] The incentive to signal then rests more on implicit *reputational* considerations of the manager. Hence, as a methodological contribution, we develop a novel framework under which counter-signaling emerges that does not rely on conventional signaling assumptions.

Our results offer new empirical predictions. The results imply that intermediate performing firms should more frequently issue earnings guidance relative to low and high-performing firms. Specifically, the probability of forecasting with respect to performance should resemble an inverted U-shape, whereby intermediate firms have the greatest likelihood

[5]The typical assumption in signaling models is that the cost of taking the action is decreasing in the sender's type. The analog of this assumption in the current setting would be that high type firms can issue guidance at a lower cost than low type firms. We find this assumption to be unsuitable in the present setting as there does not appear to be a strong justification for disclosure costs (such as proprietary or certification costs) to be *decreasing* in managerial type. While there may be indirect costs to issuing guidance, such as inducing managerial myopia, we contend that these costs would not be systematically lower for higher-type managers.

[6]While the assumption that forecast error is decreasing in ability provides a kind of sorting condition for separation, this is distinct from conventional single-crossing conditions as the direct cost of the action is the same for all types.

of issuing guidance. Ajinkya, Bhojraj, and Sengupta (2005) and Baik, Farber, and Lee (2011) document evidence partially consistent with this prediction. Ajinkya et al. (2005) find that firms with negative or zero earnings forecast less frequently, and Baik et al. (2011) document that low-ability managers tend to forecast less often. Our results imply an additional layer to this analysis, where we should observe a *decreasing* forecasting frequency in the partitioned sample of medium and well-performing firms. As preliminary evidence, we introduce the stylized fact in Figure 1, which is consistent with this prediction.

The results provide additional insights regarding the types of industries for which we should expect to find counter-signaling. A thorough discussion is included in Section 5, though we highlight the main predictions which emerge here. The results predict that the forecasting frequency of intermediate-performing firms, relative to high and low-performing firms, should be greater in industries where (i) managerial control is relatively more salient; (ii) there is greater earnings uncertainty or firm complexity; and (iii) firm performance is relatively less heterogeneous.

1.1 Related Literature

We build from Feltovich, Harbaugh, and To (2002), who develop the general theory of counter-signaling. While we share the important similarity that the receiver can potentially learn multiple pieces of information, our analysis varies from Feltovich et al. (2002), and its follow-up applications in education (Araujo et al., 2007) and portfolio management (Gervais and Strobl, 2015), in that the signal transmission is costless for the sender. Hence, we show that counter-signaling can arise in a setting without the standard single-crossing property, but rather under a less restrictive sorting condition. Moreover, the potential first signal—the forecast—provides additional information to the market (i.e., the manager's predictive ability) when the second signal, the earnings announcement, arrives. Our setting thus has an additional layer in that we examine the complementarity and interaction between different pieces of information. Furthermore, while the forecast is unbiased for all manager types, the second-order effect from the signal variance affects the manager's disclosure choice. This complements traditional signaling models which generally focus on direct effects of the signaling cost. Lastly, our setting features a rich information structure, as the market can update on potentially three pieces of information—the decision of whether or not to forecast, the forecast error, and the earnings level.

This paper is also related to the theoretical literature on earnings guidance. Trueman (1986) considers a setting where the timing of forecasts is indicative of managerial ability, thus inducing managers of high ability to forecast earlier. Our study varies in that we focus on how high ability managers can convey their type to the market by not issuing guidance. Ramakrishnan and Wen (2016) examine managerial bias in a model where the manager is probabilistically informed, and find that forecasts are optimistic in equilibrium. However, they do not consider a central feature of our setting: managerial forecast accuracy, as well as the issuance of guidance, conveys information regarding the firm's future value. Aghamolla and Hashimoto (2019) share a variant of this assumption. Their model assumes that the manager's informedness of future earnings affects the quality of the present investment. They find that earnings guidance induces myopic distortions in investment to meet the forecast. In contrast, we are principally concerned with the informational role of earnings forecast accuracy, and how highly accurate or high performing firms can withhold guidance.[7]

Moreover, a main point of departure from the extant earnings guidance literature is that the manager is able to withhold the forecast in our setting, and thus the forecasting decision itself provides information. Accordingly, our paper is related to the literature on voluntary disclosure. Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) show the influential "unraveling" result, whereby full disclosure ensues in the absence of disclosure frictions. The basic friction that prevents unraveling in our setting is a commitment to the guidance policy prior to observing the earnings signal. This has been used widely in the prior voluntary disclosure literature, such as in Diamond and Verrecchia (1991), Arya and Mittendorf (2005, 2016), Göx and Wagenhofer (2009), Rayo and Segal (2010), Arya et al. (2015), Heinle and Verrecchia (2015), Jehiel (2015), Marinovic and Sridhar (2015), Edmans, Heinle, and Huang (2016) and Li and Shi (2017).

Teoh and Hwang (1991) analyze a disclosure setting with binary types in which good

[7]Beyer (2009) considers a setting where the manager privately observes the mean and variance of the earnings distribution and must issue a forecast. She finds that the manager biases the forecast to reduce the perceived variance of earnings. The present study varies in that we do not allow for earnings management and rather consider forecasting behavior when firm performance is linked to accuracy.

firms can afford to reveal bad news as they anticipate good news in the future. Bad firms, however, disclose only good news since that may be the only good news they can reveal. In the resulting equilibrium, good firms separate from bad firms whenever bad news is revealed. Separation is obtained by relying on an exogenously assumed complementarity between the firms type and current news in producing future cash flows. Our setting differs as the incentive structure and thus the equilibrium depend on the manager's reputational considerations of her accuracy in the forecast. In addition, our setting considers a richer type space and a continuous distribution for earnings and the manager's signal. We consequently find that the equilibrium entails partial-pooling by the high-type, whereas Teoh and Hwang (1991) find full separation.

Our study is also related to the literature which examines reputational considerations in information transmission. Ottaviani and Sørensen (2006) investigate a cheap talk model where the sender has preferences over the receiver's belief of her signal precision. Their results show that this induces biased communication. The sender in our setting also has preferences for perceptions of her accuracy. However, we assume that the sender cannot manipulate the information, but rather is able to withhold the transmission. This allows the receiver (i.e., the market) to update using both the realization of the state variable (earnings), as well as the sender's decision to transmit the information.

Lastly, our paper is related to the recent literature on Bayesian persuasion, initially developed by Kamenica and Gentzkow (2011). As in the Bayesian persuasion literature, we assume that the sender determines the disclosure policy prior to observing the realization of the disclosed value. Our setting complements this literature as we assume that the disclosure choice is made when the sender has private information, which thus makes signaling a salient feature of the analysis.

The remainder of the paper is structured as follows. Section 2 introduces the model, and Section 3 presents equilibrium preliminaries. Section 4 presents our general results on counter-signaling, and considers two parameter specifications for the distribution of the forecast error. Section 5 discusses empirical implications, while Section 6 considers extensions. The final section concludes. All proofs are relegated to the Appendix.

2 Model

We develop a model with a firm manager who may choose to issue guidance regarding the firm's earnings to a market with rational risk neutral investors. We assume the manager may or may not have discretion over the firm's forecasting policy. A manager who does not have discretion must always issue a forecast, whereas a manager who has discretion can choose to provide guidance or to keep quiet.[8] If a forecast is issued, the market cannot distinguish between managers with discretion and those without it. However, investors share the common belief that the manager has discretion with probability $\rho \in (0, 1]$.

We assume that the manager privately observes her ability (type), or talent, θ. We refer to a firm with a manager of ability θ as a "θ-firm" and to that manager as a "θ-manager." We assume that there are three manager types, $\theta \in \Theta \equiv \{L, M, H\}$, which represent low, medium, and high ability, respectively. We denote the prior probability that a manager is of type θ by $p_\theta \in (0, 1)$.

The manager's ability affects the distribution of both the firm's earnings number, $e \in \mathbb{R}$, which is publicly announced after the guidance decision, and the error of a signal about future earnings, $y = e + \varepsilon$. Specifically, a manager of higher ability is more likely to produce higher earnings and a smaller forecast error. The signal y is privately observed by the manager.

We let $g(e|\theta)$ and $G(e|\theta)$ represent the probability density function (PDF) and cumulative distribution function (CDF) of earnings given ability θ, respectively. We denote by μ_θ the mean of $g(e|\theta)$. We assume that the family of densities $g(e|\theta)$ satisfies the monotone likelihood ratio property (MLRP). That is, for all $e_1 > e_0$, we have

$$\frac{g(e_1|H)}{g(e_0|H)} \geq \frac{g(e_1|M)}{g(e_0|M)} \geq \frac{g(e_1|L)}{g(e_0|L)},$$

which implies that $\mu_H \geq \mu_M \geq \mu_L$. For simplicity, we also assume that all earnings distributions have the same support regardless of the manager's ability.

We use $h(\varepsilon|\theta)$ and $H(\varepsilon|\theta)$ to represent the corresponding PDF and CDF of the forecast error given θ, respectively. We denote the support of the forecast error of a manager with talent θ by $S(\theta)$. We assume that the distribution of the forecast error is symmetric with

[8] A similar assumption is made in Acharya, DeMarzo, and Kremer (2011) and Beyer and Dye (2012).

mean 0, and the forecast of a manager with higher talent is more precise and therefore less dispersed, i.e., for all $\varepsilon_0 > \varepsilon_1 > 0$, we have

$$\frac{h(\varepsilon_1|H)}{h(\varepsilon_0|H)} \geq \frac{h(\varepsilon_1|M)}{h(\varepsilon_0|M)} \geq \frac{h(\varepsilon_1|L)}{h(\varepsilon_0|L)}.$$

Note that this implies that the support of the forecast error of a manager with higher ability is a subset of the support of the forecast error of a manager with lower ability, i.e., $S(H) \subseteq S(M) \subseteq S(L)$

Prior to the mandatory earnings announcement made by the firm, a manager who has discretion can choose to issue guidance by disclosing the signal she received, y, to the market.[9] We assume that the manager with discretion must select the guidance strategy prior to observing the signal. This is a standard assumption to prevent unraveling, and has been used in a number of voluntary disclosure settings following Diamond and Verrecchia (1991),[10] as well as in the recent literature on Bayesian persuasion (Kamenica and Gentzkow, 2011). Anecdotal evidence suggests that this assumption is consistent with actual practice, where the firm generally announces their guidance policy well in advance of the actual forecast date. For example, Google announced before its public offering in 2004 that it would not be providing earnings guidance, and has consequently not done so thereafter. Similarly, firms which initiate guidance are considered to be making a "commitment" to providing it regularly, as found in survey evidence by Graham et al. (2005).[11] We may alternatively model this as the manager's observable decision to design an information system that produces a predictive signal of earnings, y, in which case, any manager who chooses to do so would have to disclose as there is no additional friction that prevents unraveling.

We denote the manager's choice of guidance strategy by $\phi \in \{F, NF\}$, where F denotes issuing a forecast and NF denotes withholding the forecast. If the manager commits to forecast, the signal y is made publicly observable through a report. Otherwise, if the manager

[9] We assume that the manager simply discloses the signal y if she chooses to issue guidance. Alternatively, the manager could disclose $E(e|y)$ without qualitatively affecting the results, however, this makes the algebra far more cumbersome.

[10] See also Arya and Mittendorf (2005), Göx and Wagenhofer (2009), Rayo and Segal (2010), Heinle and Verrecchia (2015), Jehiel (2015), Marinovic and Sridhar (2015), and Li and Shi (2017).

[11] Several other studies in the empirical literature have also found evidence consistent with a managerial commitment to issuing guidance (e.g. Gibbins et al., 1990; Bamber et al., 2010).

decides not to forecast, there is no signal, denoted by $\emptyset$.

We denote the underlying value of the firm with a manager of type θ by V_θ, where $V_H > V_M > V_L = 0$. To simplify notation, we assume that earnings are distributed as dividends in the same stage as the earnings announcement.[12] Thus, one can simply think of the value of the firm as reflecting the discounted sum of future earnings whose distribution depends on the manager's type θ.

We assume that the manager's utility, denoted by u, depends on the firm's market price, P, which is equal to the market's expectation of firm value, i.e.,

$$u = \mathbb{E}(V_\theta|\Omega),$$

where Ω denotes the market's information set, which includes the disclosure decision, the earnings forecast (if released), and the earnings announcement. Alternatively, we could rather have the manager's utility depend on the market's belief of her ability. This alternative assumption would not affect the results.

Overall, the sequence of events is the following:

Stage 1: The manager privately observes her ability θ and whether she has discretion.

Stage 2: If the manager has discretion, she choses a guidance strategy, $\phi \in \{F, NF\}$.

Stage 3: The manager observes a signal $y = e + \varepsilon$. If $\phi = F$, or if the manager does not have discretion, the signal y is made publicly observable in the form of earnings guidance. If $\phi = NF$, the manager does not provide earnings guidance, denoted by $\emptyset$.[13]

Stage 4: A mandatory earnings report, $a = e$, is released by the firm. Earnings are distributed as dividends and the market prices the firm.

[12]This does not impact any of the results, but simplifies notation, as it allows us to omit the current earnings in the market price calculation.

[13]Alternatively, we can assume that the manager only observes y if she decides to set up a predictive information system that generates such a signal. This alternative specification does not require ex post commitment not to disclose.

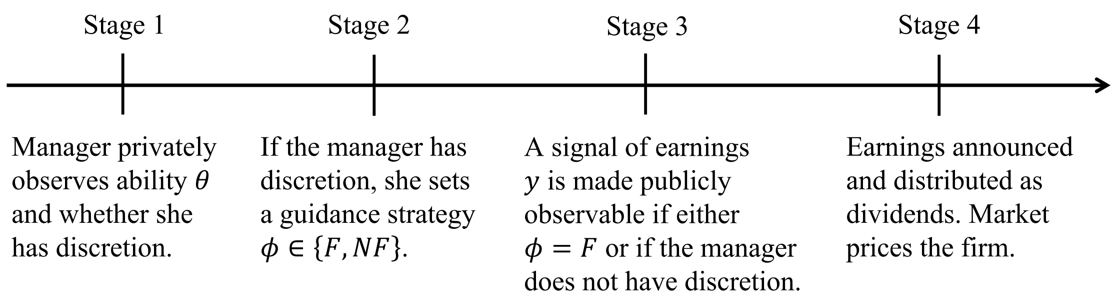

Figure 2: Timeline

3 Equilibrium Preliminaries

The equilibrium concept we employ is Perfect Bayesian Equilibrium and we focus on pure strategy equilibria. In equilibrium, a manager with discretion chooses whether to provide an earnings forecast in light of the potential impact on the market price, which is determined by the market's rational expectations about the manager's earnings guidance strategy. The equilibrium market pricing rule is represented by the function $P : \mathbb{R} \times (\mathbb{R} \cup \{\emptyset\}) \to \mathbb{R}$, where $P(e, \varepsilon)$ is the market price of the firm for a mandatory earnings announcement e and an earnings forecast error ε. Similarly, $P(e, \emptyset)$ is the market price of the firm for an earnings report e, and no issuance of guidance. We also use the function $\hat{\Phi} : \Theta \to \{F, NF\}$ to denote the market's conjecture of the manager's guidance strategy, and the function $\hat{P} : \mathbb{R} \times (\mathbb{R} \cup \{\emptyset\}) \to \mathbb{R}$ to represent the manager's conjecture of the market price P. We define the equilibrium as follows:

Definition 1. The equilibrium is defined as a vector $(\Phi, \hat{\Phi} : \Theta \to \{F, NF\}, P, \hat{P} : \mathbb{R} \times (\mathbb{R} \cup \{\emptyset\}) \to \mathbb{R})$, which satisfies the following conditions:

(i) The manager's earnings forecast strategy is optimal given the manager's anticipation of the market price, i.e.,

$$\Phi(\theta) \in \arg \max_{\phi \in \{F, NF\}} \mathbb{E}\left[\mathbb{I}_{\phi=F} \hat{P}(e, \varepsilon) + (1 - \mathbb{I}_{\phi=F}) \hat{P}\left(e, \emptyset\right) | \theta\right], \tag{1}$$

where $\mathbb{I}_{\phi=F}$ is an indicator function which is equal to 1 if $\phi = F$ and 0 otherwise.

(ii) The pricing rule is sequentially rational. That is, the market price is equal to the expected value of the firm, conditional on the public information and the conjectured manager's guidance strategy.

(iii) All agents have rational expectations regarding each other's behavior, i.e., $\Phi(\theta) = \hat{\Phi}(\theta)$ and $P(e, \cdot) = \hat{P}(e, \cdot)$.

3.1 Market Price

We first investigate how the pricing rule $P(e, \varepsilon)$ is determined given the market's beliefs of the manager's guidance strategy. Define $\hat{\Theta}_F := \left\{\theta | \hat{\Phi}(\theta) = F, \theta \in \Theta\right\}$ to be the set of θ-managers with discretion that the market believes choose to forecast. Correspondingly, $\Theta \backslash \hat{\Theta}_F = \left\{\theta | \hat{\Phi}(\theta) = NF, \theta \in \Theta\right\}$ is the set of θ-managers with discretion that, the market believes, choose not to forecast. We denote the market's beliefs of the manager's type by $\gamma(\theta|\cdot)$. Specifically, $\gamma(\theta|\cdot)$ is the probability that the manager is of type θ conditional on all public information available to the market after earnings are released. Since both the earnings level and the forecast error depend on the manager's ability, the market must update about θ using both pieces of information (if observed), including the manager's (conjectured) guidance strategy. The posterior distribution of the manager's ability θ given both the earnings announcement and the forecast error is given by:

$$\gamma\left(\theta | e, \varepsilon; \hat{\Theta}_F\right) = \frac{\mathbb{I}_{\theta \in \Theta \backslash \hat{\Theta}_F}(1-\rho) p_\theta g(e|\theta) h(\varepsilon|\theta) + \mathbb{I}_{\theta \in \hat{\Theta}_F} p_\theta g(e|\theta) h(\varepsilon|\theta)}{\sum_{\theta' \in \Theta \backslash \hat{\Theta}_F}(1-\rho) p_{\theta'} g(e|\theta') h(\varepsilon|\theta') + \sum_{\theta' \in \hat{\Theta}_F} p_{\theta'} g(e|\theta') h(\varepsilon|\theta')}, \tag{2}$$

where $(1-\rho) p_\theta g(e|\theta) h(\varepsilon|\theta)$ is the probability that a manager has no discretion, has ability θ, generates earnings e, and discloses guidance $y = e + \varepsilon$. Similarly, $p_\theta g(e|\theta) h(\varepsilon|\theta)$ is the probability that a manager has ability θ (with discretion or not), generates earnings e, and discloses guidance $y = e + \varepsilon$. Hence, the market price when guidance is issued is given by:

$$P(e, \varepsilon) = \sum_{\theta \in \Theta} \gamma\left(\theta | e, \varepsilon; \hat{\Theta}_F\right) V_\theta. \tag{3}$$

Investors believe that only managers with discretion and with ability θ in the set $\Theta \backslash \hat{\Theta}_F$ choose not to issue guidance. The posterior belief of the manager's ability given the announcement

e and the absence of a forecast is given by:

$$\gamma\left(\theta|e,\emptyset;\hat{\Theta}_F\right)=\frac{\mathbb{I}_{\theta\in\Theta\setminus\hat{\Theta}_F}\rho p_\theta g(e|\theta)}{\sum_{\theta'\in\Theta\setminus\hat{\Theta}_F}\rho p_{\theta'}g(e|\theta')},\tag{4}$$

where $\rho p_\theta g(e|\theta)$ is the probability that a manager has discretion, is of type θ, and announces earnings e. Therefore, the market price for a firm without an earnings forecast is given as:

$$P\left(e,\emptyset\right)=\mathbb{E}\left[v|e,NF\right]=\sum_{\theta\in\Theta}\gamma\left(\theta|e,\emptyset;\hat{\Theta}_F\right)V_\theta.\tag{5}$$

3.2 Manager's Guidance Strategy

We now analyze the manager's optimal forecasting strategy given her anticipation of the market price. The manager's objective is to maximize the market price of the firm after the earnings announcement, and hence she decides to issue guidance if this weakly improves the price in expectation. Given the manager's expectation of the market pricing rule $\hat{P}\left(\cdot,\cdot\right)$, by issuing guidance, a manager with ability $\theta\in\Theta$ receives the following expected payoff:

$$\mathbb{E}_{\hat{P}}\left[u(F|\theta)\right]=\mathbb{E}\left[\hat{P}\left(e,\varepsilon\right)|\theta\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{P}(e,\varepsilon)g(e|\theta)h(\varepsilon|\theta)\,\mathrm{d}e\,\mathrm{d}\varepsilon,\tag{6}$$

where $g(e|\theta)h(\varepsilon|\theta)$ is the probability that a manager of type θ generates earnings e and forecasts with an error ε. In contrast, by choosing not to issue guidance, the manager's expected payoff is given by:

$$\mathbb{E}_{\hat{P}}\left[u(NF|\theta)\right]=\mathbb{E}\left[\hat{P}\left(e,\emptyset\right)|\theta\right]=\int_{-\infty}^{\infty}\hat{P}\left(e,\emptyset\right)g(e|\theta)\,\mathrm{d}e.\tag{7}$$

Thus, a manager with discretion and talent θ chooses to forecast if and only if $\mathbb{E}_{\hat{P}}\left[u(F|\theta)\right]\geq\mathbb{E}_{\hat{P}}\left[u(NF|\theta)\right]$.

3.3 Equilibrium Property

Due to rational expectations, in equilibrium all conjectured strategies must be equal to the equilibrium strategies. Hence, $\hat{\Phi}\left(\theta\right)=\Phi^*\left(\theta\right)$ and $\hat{P}\left(e,\cdot\right)=P^*\left(e,\cdot\right)$, where $\Phi^*\left(\theta\right)$

and $P^*(e,\cdot)$ represent the equilibrium forecast strategy and the equilibrium market price, respectively. We denote by Θ_F^* as the subset of managers who choose to forecast in equilibrium, i.e., $\Theta_F^* := \left\{\theta | \Phi^*(\theta) = F, \theta \in \Theta\right\}$. From equations (3) and (6), the expected payoff of a manager with discretion who chooses to forecast in equilibrium is:

$$\mathbb{E}\left[u(F|\theta)|\Theta_F^*\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{\theta'\in\Theta}\gamma\left(\theta'|e,\varepsilon;\Theta_F^*\right)V_{\theta'}g(e|\theta)h(\varepsilon|\theta)\,\mathrm{d}e\,\mathrm{d}\varepsilon. \tag{8}$$

Similarly, from equations (5) and (7), the expected payoff of the manager with talent $\theta \in \Theta$ who chooses not to forecast in equilibrium is:

$$\mathbb{E}\left[u(NF|\theta)|\Theta_F^*\right] = \int_{-\infty}^{\infty}\sum_{\theta'\in\Theta}\gamma\left(\theta'|e,\emptyset;\Theta_F^*\right)g(e|\theta)V_{\theta'}\,\mathrm{d}e. \tag{9}$$

We denote by $\Delta\left(\theta|\Theta_F^*\right)$ the difference in the expected payoff that a manager with discretion obtains from issuing guidance over remaining silent, i.e., $\Delta\left(\theta|\Theta_F^*\right) = \mathbb{E}\left[u(F|\theta)|\Theta_F^*\right] - \mathbb{E}\left[u(NF|\theta)|\Theta_F^*\right]$. An equilibrium is thus characterized by a set $\Theta_F^* \subseteq \Theta$, such that for all $\theta \in \Theta_F^*$, $\Delta\left(\theta|\Theta_F^*\right) \geq 0$, and for all $\theta \in \Theta\backslash\Theta_F^*$, $\Delta\left(\theta|\Theta_F^*\right) < 0$.

4 Counter-Signaling Equilibrium

In this section, we derive the general set of equilibria that are feasible. We first define a *counter-signaling* equilibrium in our setting as follows:

Definition 2. A *counter-signaling* equilibrium is an equilibrium in which, among the managers with discretion, only the medium-ability manager chooses to issue guidance, while the low-ability and high-ability managers with discretion only report the realized earnings, i.e., $\Theta_F^* = \{M\}$.

As we see above, we define counter-signaling as an equilibrium where, of those managers who have discretion, only the intermediate manager M issues guidance. We now present a preliminary result that establishes monotonicity in the market's pricing rule.

Lemma 1. *Given any belief by the market, the price for firms who issue guidance, $P(e,\varepsilon)$, is weakly increasing in earnings e and weakly decreasing in the absolute value of the forecast* *error $|\varepsilon|$, i.e.,* $\frac{\partial P(e,\varepsilon)}{\partial e} \geq 0$, $\frac{\partial P(e,\varepsilon)}{\partial \varepsilon} \geq 0$ *for* $\varepsilon \leq 0$, *and* $\frac{\partial P(e,\varepsilon)}{\partial \varepsilon} \leq 0$ *for* $\varepsilon > 0$. *The market price for firms without an earnings forecast is increasing in earnings e, i.e.,* $\frac{\partial P\left(e,\emptyset\right)}{\partial e} \geq 0$.

Lemma 1 states that, when guidance is issued, the price is higher when the market observes a higher earnings level, e, and a lower absolute value of the forecast error, $|\varepsilon|$. Likewise, if there is no earnings forecast, the market price is higher with a higher reported earnings level e.

We now establish two of our main results. We examine the set of equilibria that are feasible in two scenarios: partial discretion ($\rho < 1$) and full discretion ($\rho = 1$). We begin with the partial discretion case, since, as we will see, the full discretion case builds on these results. The following theorem states that, when the manager does not have full discretion, then the only feasible equilibria involve either partial separation (as counter-signaling) or full pooling.

Theorem 1. *Assume $\rho \in (0,1)$. In any equilibrium, $\Theta_F^* \in \left\{\Theta, \{M\}\right\}$. That is, the feasible set of equilibria consists of counter-signaling and full pooling.*

Theorem 1 states that the *only* possible partial-separating equilibrium is the counter-signaling equilibrium. We note that Theorem 1 does not establish existence or uniqueness, but rather characterizes the set of feasible equilibria. We find this to be a quite general result, as it holds for general distributions of the error ε and of earnings e that satisfy natural monotonicity assumptions. We also do not impose any conditions on the distribution of the discretized type space. As this is a general analysis, in order to precisely characterize the conditions under which counter-signaling exists and is the unique equilibrium, we must specify a functional form for the distribution of ε. This analysis is developed in Sections 4.1 and 4.2.

Among the two equilibria specified in Theorem 1, one is the full-pooling equilibrium, $\Theta_F^* = \Theta$, in which every manager type issues a forecast. This equilibrium is typical of signaling settings and is supported by the off-equilibrium-path beliefs that any manager who does not issue a forecast is of a low type.

The second equilibrium in the feasible set is the counter-signaling equilibrium, $\Theta_F^* = \{M\}$. For expositional ease, we omit qualifiers of "discretion" and "no discretion," as it will be clear from the context that managers who choose to issue or withhold guidance have discretion. We first discuss the underlying intuition for why a counter-signaling equilibrium emerges, and then discuss why no other partially separating equilibrium is feasible. With both a forecast and the earnings announcement, the market receives considerable information with which to make an assessment of the manager's ability. Consequently, low types wish to withhold as much information from the market as possible. Conversely, the medium and high ability managers have less reservations in releasing information regarding their ability. Suppose the M-manager issues a forecast in equilibrium. In turn, the high type may choose to either issue guidance, in which case she is pooled with M and the set of managers with no discretion, or she may choose to keep quiet, in which case she is pooled with L. The high type prefers the latter, as her earnings announcement is relatively more discriminating of her type when the only other possibility is that she is an L-manager. Hence, the H-manager also keeps quiet. The economic force driving the result is that the high-ability manager can more easily distinguish herself when she is ex ante pooled with the lowest type, even though she can costlessly reveal more information to the market. This argument thus relies on indirect consequences to the market's Bayesian updating rather than on direct cost-saving by the high ability manager.[14]

We now discuss why no other equilibrium is possible. First, we cannot have an equilibrium where the L-manager is the only type with discretion that issues guidance (i.e., $\Theta_F^* \neq \{L\}$), or the only type that withholds guidance (i.e., $\Theta_F^* \neq \{M, H\}$). Indeed, if this is the case, the L-manager is the only type separating. The L-manager thus has a strictly profitable deviation of pooling with the medium and high types, as this set generates a strictly higher pooled value for the L-manager in expectation. Second, an H-manager cannot be the only type withholding guidance in equilibrium, i.e., $\Theta_F^* \neq \{L, M\}$. In this case, as in the previous one, the low and medium ability managers have an incentive to deviate from forecasting to keeping quiet in order to pool with the high type manager.

Third, an M-manager cannot be the only type withholding an earnings forecast in

[14]For completeness, we note that the L-manager does not wish to deviate, as this would then provide the market more information of her ability and the set of pooled types (M-managers with discretion and all no-discretion managers) is strictly worse than the set of H-managers with discretion in terms of ex ante expected value. Similarly, M-managers do not wish to deviate as they can distinguish themselves best by being the only type with discretion who issues a forecast.

equilibrium, i.e., $\Theta_F^* \neq \{L, H\}$. The argument is more subtle in this case. Assume that such an equilibrium holds. Since all firms without a forecast are M-firms, any firm without an earnings forecast is priced as an M-firm. For the equilibrium to hold, L-managers must prefer to forecast. This in turn implies that, by providing a forecast, an L-manager must be able to obtain a higher price than that of a perfectly identified M-firm. Moreover, since M-managers can provide more precise forecasts than L-managers, the expected market price of a firm with an M-manager should be higher than that of a firm with an L-manager if both issue guidance. However, this is a contradiction because, in that case, the M-manager should be better off by providing forecast, as she would expect to obtain a higher price than that of a non-forecasting M-firm.

Fourth, it cannot be the case that all managers (with discretion) withhold guidance in equilibrium, i.e., $\Theta_F^* \neq \emptyset$. In this case, an H-manager has an incentive to deviate to providing guidance, since, by forecasting jointly with all no-discretion managers, a more precise forecast serves as an additional signal to differentiate H-managers from L and M-managers. Similarly, we cannot have an equilibrium where only H-managers issue guidance, i.e., $\Theta_F^* \neq \{H\}$. In this case, an M-manager has an incentive to deviate to forecasting. Indeed, withholding guidance leads to pooling with L-managers. By forecasting, however, the M-manager can separate herself from an L-firm in two ways. First, a more precise forecast serves as an additional signal to differentiate M-managers from L-managers. Second, forecasting precision aside, by issuing guidance she pools with all of the discretion H-managers and no-discretion managers, which constitute a set of managers with a higher expected ability.

We now consider the case with full discretion, $\rho = 1$. Analogous to Theorem 1, we provide a general result which characterizes the set of feasible equilibria in the case in which the manager has discretion with probability one. With full discretion, a support for the forecast error that changes with the manager's type plays a crucial role in the market's belief updating, allowing for additional feasible equilibria. In the following theorem, we build on the analysis from Theorem 1:

Theorem 2. *Assume the manager has full discretion, i.e., $\rho = 1$, and the supports for the forecast errors are such that $S(H) \subset S(M) \subset S(L)$. The set of feasible equilibria is given as $\Theta_F^* \in \big\{\emptyset, \Theta, \{H\}, \{M\}\big\}$.*

Theorem 2 states that if all manager types have discretion, only four guidance strategies can be supported in equilibrium, $\Theta_F^* \in \{\emptyset, \Theta, \{H\}, \{M\}\}$. Among these cases, two are full-pooling equilibria, $\Theta_F^* \in \{\emptyset, \Theta\}$, where either every type forecasts or every type keeps quiet. There are only two possible partial separating equilibria, $\Theta_F^* \in \{\{H\}, \{M\}\}$, where $\Theta_F^* = \{M\}$ represents the counter-signaling equilibrium. Notice that, in contrast with Theorem 1, full discretion makes two additional equilibria feasible: $\Theta_F^* \in \{\emptyset, \{H\}\}$. The pooling equilibrium $\Theta_F^* = \emptyset$ is now feasible because, with full discretion, a forecast is now an off-equilibrium-path action. Thus, this pooling equilibrium can be sustained by the market belief that any forecasting manager is of type L. The partial separating equilibrium $\Theta_F^* = \{H\}$ is also sustained by off-path beliefs. Suppose, for instance, that the market observes a forecast with an error outside the high type's error support, i.e., $\varepsilon \notin S(H)$. Since this is an off-equilibrium-path event, we can assume that the market believes that the forecast is issued by the L-manager. Thus, if the H-manager's forecast is sufficiently precise, the L and M-managers prefer not to deviate as they risk being identified as an L-manager.

To illustrate the results regarding equilibrium feasibility, and to determine precise conditions under which a counter-signaling equilibrium exists and is unique, we must specify a functional form for the distribution of the forecast error. We first illustrate the case of partial discretion ($\rho < 1$) by assuming a normally distributed forecast error, and then analyze the case of full discretion ($\rho = 1$) with a uniform specification. In both cases, we continue to use a general distribution for earnings, e, which satisfies the conditions outlined in Section 2. We show that the conditions for the existence of a counter-signaling equilibrium are largely qualitatively similar under the two cases.

4.1 Normally Distributed Forecast Error

We first consider a setting with a normally distributed forecast error, i.e., $\varepsilon|\theta \sim N\left(0, \sigma_\theta^2\right)$, with $\sigma_L^2 > \sigma_M^2 > \sigma_H^2 > 0$, which implies that the signal precision is increasing in ability. The price for a non-guidance firm in a counter-signaling equilibrium, based on earnings announcement e and the fact that the firm did not issue guidance, is given by:

$$P\left(e,\emptyset\right)=\mathbb{E}\left[v|e,\emptyset\right]=\frac{p_{L}g\left(e|L\right)V_{L}+p_{H}g\left(e|H\right)V_{H}}{p_{L}g\left(e|L\right)+p_{H}g\left(e|H\right)}=\frac{V_{H}}{\frac{p_{L}g\left(e|L\right)}{p_{H}g\left(e|H\right)}+1}. \tag{10}$$

Note that the firm value associated with an L-manager does not appear in the numerator as $V_L = 0$. Next, we denote by $\zeta\left(\theta\right)$ the θ-manager's ex ante expectation of the market's belief that she is of type H after withholding guidance. Formally,

$$\zeta\left(\theta\right)=\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)}{\frac{p_{L}g\left(e|L\right)}{p_{H}g\left(e|H\right)}+1}\,\mathrm{d}e.$$

In other words, with only the prior information about earnings, a manager of type θ expects the market to assign a probability $\zeta(\theta)$ to the event that she is of type H if she does not issue guidance. We first derive a preliminary result regarding $\zeta(\theta)$:

Lemma 2. *$\zeta(\theta)$ is increasing in θ, i.e., $\zeta(L)<\zeta(M)<\zeta(H)$.*

Lemma 2 states that managers of higher ability have a higher ex ante expectation of being perceived as an H-manager after not issuing a forecast. This follows from the fact that, without guidance, the earnings announcement is the only signal that the market observes. Since a higher-ability manager is more likely to obtain high earnings, she thus shares an ex ante higher expectation of eventually being perceived as type H.

Similarly, the price for a forecasting firm in a counter-signaling equilibrium is given by:

$$\begin{aligned}P(e,\varepsilon)&=\mathbb{E}[v|e,\varepsilon,F]\\&=\frac{(1-\rho)\left(p_{L}g\left(e|L\right)h\left(\varepsilon|L\right)V_{L}+p_{H}g\left(e|H\right)h\left(\varepsilon|H\right)V_{H}\right)+p_{M}g\left(e|M\right)h\left(\varepsilon|M\right)V_{M}}{(1-\rho)\left(p_{L}g\left(e|L\right)h\left(\varepsilon|L\right)+p_{H}g\left(e|H\right)h\left(\varepsilon|H\right)\right)+p_{M}g\left(e|M\right)h\left(\varepsilon|M\right)}.\end{aligned}$$

We also define $\varphi_M(\theta)$ and $\varphi_H(\theta)$ as the θ-manager's ex ante expectation of being perceived as type M and type H correspondingly after *issuing* guidance.[15] Therefore, by the definition of $\Delta(\theta|\Theta_F^*)$, in a counter signaling equilibrium where $\Theta_F^* = \{M\}$, the manager's expected payoff difference from forecasting over keeping quiet is given by:

$$\Delta\left(\theta|\{M\}\right) = \varphi_M\left(\theta\right)V_M + \left[\varphi_H\left(\theta\right) - \zeta\left(\theta\right)\right]V_H. \tag{11}$$

Therefore, we must have $\Delta(M|\{M\}) \geq 0$, $\Delta(L|\{M\}) \leq 0$, and $\Delta(H|\{M\}) \leq 0$ to sustain a counter-signaling equilibrium. That is, the low and high types do not have a profitable deviation to forecasting, as this would lead to a lower payoff, and analogously the medium type does not have a profitable deviation to withholding. We characterize existence and uniqueness of the counter-signaling in the following:

Proposition 1. *Assume $\rho \in (0,1)$ and errors are normally distributed. The counter-signaling equilibrium uniquely exists if and only if:*
(i) $\frac{\zeta(M)-\varphi_H(M)}{\varphi_M(M)} \leq \frac{V_M}{V_H} \leq \min\left\{\frac{\zeta(H)-\varphi_H(H)}{\varphi_M(H)}, \frac{\zeta(L)-\varphi_H(L)}{\varphi_M(L)}\right\}$*;*
(ii) ρ is sufficiently large;
(iii) $\frac{\sigma_L}{\sigma_M}$ is sufficiently large; and
(iv) $\frac{\sigma_M}{\sigma_H}$ is sufficiently small.

Proposition 1 states that the counter-signaling equilibrium uniquely exists if four conditions are met. Condition (i) characterizes the *valid region* of the ratio V_M/V_H under which counter-signaling arises in equilibrium. On one hand, V_M/V_H needs to be sufficiently large. This is natural as otherwise M-managers would find it profitable to withhold guidance in an attempt to pool with H-managers. Stated differently, an extremely high V_H relative to V_M entices pooling, even if this raises the possibility that the M-manager is misidentified

[15] Specifically, $\varphi_M(\theta)$ is defined as:

$$\varphi_M\left(\theta\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{g\left(e|\theta\right)h\left(\varepsilon|\theta\right)}{(1-\rho)\left(\frac{p_L g\left(e|L\right)h\left(\varepsilon|L\right)}{p_M g\left(e|M\right)h\left(\varepsilon|M\right)} + \frac{p_H g\left(e|H\right)h\left(\varepsilon|H\right)}{p_M g\left(e|M\right)h\left(\varepsilon|M\right)}\right) + 1}\,\mathrm{d}e\,\mathrm{d}\varepsilon,$$

and $\varphi_H(\theta)$ is defined as:

$$\varphi_H\left(\theta\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{g\left(e|\theta\right)h\left(\varepsilon|\theta\right)}{\frac{p_L g\left(e|L\right)h\left(\varepsilon|L\right)}{p_H g\left(e|H\right)h\left(\varepsilon|H\right)} + 1 + \frac{1}{(1-\rho)}\frac{p_M g\left(e|M\right)h\left(\varepsilon|M\right)}{p_H g\left(e|H\right)h\left(\varepsilon|H\right)}}\,\mathrm{d}e\,\mathrm{d}\varepsilon.$$

as type L. On the other hand, V_M/V_H cannot be too large, as otherwise L-mangers would prefer to forecast and receive the payoff $\varphi_M(\theta)V_M + \varphi_H(\theta)V_H$ by pooling with M-managers.

Condition (ii) requires ρ to be sufficiently large but still smaller than one. A small ρ implies that the manager is likely to lack discretion. In that case, a small forecast error is indicative that the manager is of type H without discretion. Thus, an H-manager with discretion prefers to disclose. Note that $\rho < 1$ is necessary for the counter-signaling equilibrium to exist under normally distributed forecast errors. If $\rho = 1$, all managers have discretion and, in a counter-signaling equilibrium, the mere act of forecasting would perfectly identify the manager as being of type M. However, since forecasting is costless for all manager types and the forecast errors have full support, the L-manager could perfectly and costlessly mimic the M-manager by issuing a forecast. If instead there is some likelihood that some managers lack discretion, regardless of how small, but at least positive, a deviation from type L allows for the possibility that the market assesses this manager to be of type L based on the forecast error and earnings level.[16]

Conditions (iii) and (iv) pertain to the forecasting errors. We see that σ_L/σ_M must be sufficiently large, as otherwise type L wishes to forecast in order to pool with M types. Similarly, σ_M/σ_H must be small enough to prevent type H from finding forecasting profitable, as she is more likely to be misidentified as an M-manager.

We now analyze the valid region of V_M/V_H in Condition (i) using numerical examples and show that the valid region is non-empty for sufficiently large σ_L or sufficiently large σ_H. In Figure 3 we see that the valid region expands as σ_L increases. This is natural as a relatively lower mimicking incentive from the L-manager allows for a greater degree of freedom of the difference between V_M and V_H. Likewise, in Figure 4, the valid region expands as σ_H increases. This occurs as forecasting by the H-manager becomes less effective for separation when σ_H is relatively high.

[16] Additionally, this equilibrium does not rely on any specification of off-path beliefs, as any realization for the forecast error can occur on the equilibrium path. Allowing for a small probability of non-discretion leads market participants to incorporate the earnings announcement in their updated beliefs, regardless of whether or not the manager provides a forecast — a feature which is arguably consistent with practice.

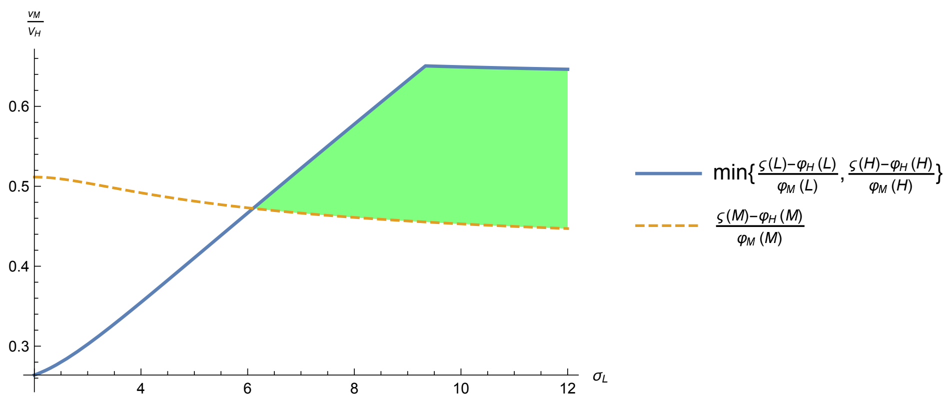

Figure 3: Changes in the valid region of V_M/V_H given changes in σ_L, where $\rho = 1/2, e|\theta \sim N(\mu_\theta, 1)$, $\mu_H = 2, \mu_M = 1, \mu_L = 0, \sigma_H = 1, \sigma_M = 2, p_L = p_H = 1/4$, and $p_M = 1/2$.

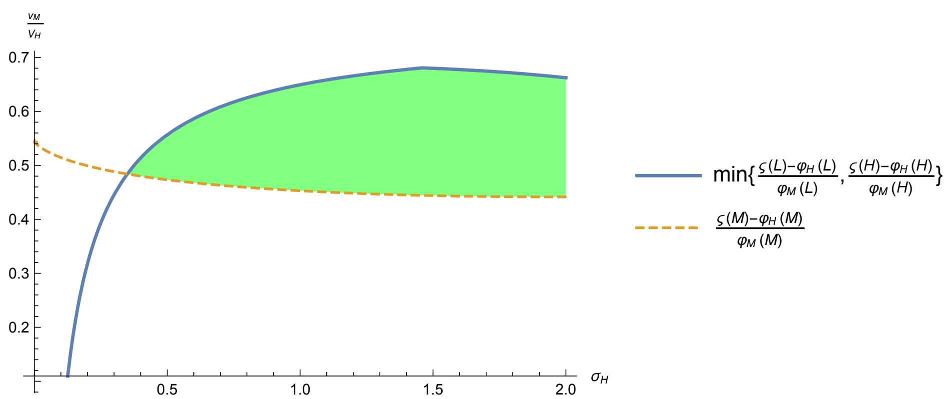

Figure 4: Changes in the valid region of V_M/V_H given changes in σ_H, where $\rho = 1/2, e|\theta \sim N(\mu_\theta, 1)$, $\mu_H = 2, \mu_M = 1, \mu_L = 0, \sigma_M = 2, \sigma_L = 10, p_L = p_H = 1/4$, and $p_M = 1/2$.

4.2 Uniformly Distributed Forecast Error with Full Discretion

We now consider the case in which the forecast errors are uniformly distributed, i.e., $\varepsilon|\theta \sim U[-b_\theta, b_\theta]$, where $b_L > b_M > b_H > 0$, and proceed to showing existence and uniqueness of the counter-signaling equilibrium when the manager has discretion with certainty, $\rho = 1$. In this case, M-managers separate perfectly from managers with ability L and H by providing a forecast, while L-managers are conditionally pooled with H-managers. Hence, in equilibrium, upon observing a forecast, the market believes that the manager is of type M, regardless of the earnings announcement. Correspondingly, the price for forecasting firms is given as $P(e, \varepsilon) = V_M$, while the price for non-issuing firms is given by equation (10).

In order for counter-signaling to be an equilibrium, no type must have an incentive to deviate. If an M-manager deviates to withholding guidance, the market assigns a price of $P(e, \emptyset)$ after observing the firm's earnings e. It is slightly more complicated if a manager with ability L or H deviates to providing guidance. Specifically, the forecast error of an L-manager may be outside of $[-b_M, b_M]$, which is not observed on the equilibrium path. To complete the equilibrium, we specify the following off-path beliefs: Upon observing $\varepsilon \notin [-b_M, b_M]$, the market believes that the manager is of type L with probability one. That is, an L-manager reveals her true ability if her guidance is so imprecise that the gap between the forecast and the realized earnings is larger than b_M. Lastly, an H-manager's deviation to providing guidance would lead to a market price of that for an M type. Overall, the market price for a firm that has issued guidance is:

$$P(e, \varepsilon) = \begin{cases} 0 & \text{if } |\varepsilon| > b_M; \\ V_M & \text{if } |\varepsilon| \leq b_M. \end{cases} \tag{12}$$

Therefore, in a counter-signaling equilibrium where $\Theta_F = \{M\}$, the difference in the expected payoff that a manager with discretion obtains from issuing guidance over keeping quiet is given by:

$$\Delta\left(\theta|\{M\}\right) = \min\left\{1, \frac{b_M}{b_\theta}\right\} V_M - \zeta(\theta) V_H, \quad \theta \in \{L, M, H\}. \tag{13}$$

As in the previous case, in order to sustain a counter-signaling equilibrium, there cannot be profitable deviations, i.e., $\Delta(M|\{M\}) \geq 0$, $\Delta(L|\{M\}) \leq 0$, and $\Delta(H|\{M\}) \leq 0$. The following lemma determines conditions under which this holds:

Lemma 3. *There exists an equilibrium in which managers of type M provide guidance, while managers of type L and H do not provide guidance if and only if*

$$\zeta\left(M\right) \leq \frac{V_M}{V_H} \leq \min\left\{\frac{b_L}{b_M}\zeta\left(L\right), \zeta\left(H\right)\right\}, \tag{14}$$

where the valid region of the ratio $\frac{V_M}{V_H}$ is non-empty if and only if $\frac{b_L}{b_M} \geq \frac{\zeta(L)}{\zeta(M)}$.

Condition (14) of Lemma 3 is immediately determined from the non-deviation conditions. The remaining question is when these aforementioned conditions can be simultaneously satisfied. As we have shown in Lemma 2, $\zeta\left(L\right) < \zeta\left(M\right) < \zeta\left(H\right)$ which implies that the region where the conditions are valid is non-empty if and only if $\frac{b_L}{b_M} \geq \frac{\zeta(L)}{\zeta(M)}$. This holds only when b_L is sufficiently large relative to b_M. In other words, the valid interval of V_M/V_H for the existence of a counter-signaling equilibrium is non-empty if and only if the forecast precision of an L-manager is sufficiently low relative to an M-manager such that the L-manager cannot easily mimic the M type by issuing a forecast.

As shown in Theorem 2, the counter-signaling equilibrium is not the only possible partial-pooling equilibrium when all managers have discretion. Another possible equilibrium is that only H-managers provide guidance. In order to determine uniqueness of the counter-signaling equilibrium, we must examine the conditions under which $\Theta_F^* = \{H\}$ exists.

Lemma 4. *There is an equilibrium such that $\Theta_F^* = \{H\}$ if and only if:*

$$\frac{V_H}{V_M} \leq \min\left\{\frac{b_M}{b_H}\int_{-\infty}^{\infty}\frac{g\left(e|M\right)}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1}\,\mathrm{de}, \frac{b_L}{b_H}\int_{-\infty}^{\infty}\frac{g\left(e|L\right)}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1}\,\mathrm{de}\right\}.$$

Lemma 4 states the conditions under which the alternative equilibrium exists. Note that the interval of V_M/V_H above is non-empty if and only if b_M/b_H is sufficiently large, or equivalently, if b_H/b_M is small enough. When b_H/b_M is small, issuing guidance is a powerful signal for H-managers with discretion to differentiate themselves from L or M-managers, and hence they prefer to forecast. Lemma 4 is helpful in characterizing the *unique* existence of the counter-signaling equilibrium, which is shown in the following result:

Proposition 2. *Assume $\rho = 1$ and errors are uniformly distributed. The counter-signaling equilibrium uniquely exists if and only if:*
(i) $\zeta(M) \leq \frac{V_M}{V_H} \leq \min\left\{\frac{b_L}{b_M}\zeta(L), \zeta(H)\right\}$;
(ii) $\frac{b_L}{b_M} \geq \frac{\zeta(L)}{\zeta(M)}$; *and*
(iii) $\frac{b_H}{b_M} > \frac{V_M}{V_H} \min\{\int_{-\infty}^{\infty} \frac{g(e|M)}{\frac{p_L g(e|L)}{p_M g(e|M)}+1} \mathrm{d}e, \frac{b_L}{b_M}\int_{-\infty}^{\infty} \frac{g(e|L)}{\frac{p_L g(e|L)}{p_M g(e|M)}+1} \mathrm{d}e\}$.

We see from Proposition 2 that the counter-signaling equilibrium uniquely exists under certain conditions. Conditions (i) and (ii) ensure the existence of the counter-signaling equilibrium, following Lemma 3. Condition (iii) rules out the equilibrium $\Theta_F^* = \{H\}$, following Lemma 4. We note that both of the terms in the right-hand side of condition (iii), within the minimizer, are decreasing in p_L, the prior probability that $\theta = L$. Hence, condition (iii) implies that p_L, or the ex ante pool of low type managers, must be sufficiently high. This is somewhat surprising, as one would expect that the H-manager has a greater incentive to forecast to separate from the low type when p_L is high. However, the medium type also shares this greater incentive for separation from L as p_L increases, thus increasing the M-manager's incentive to forecast. Consequently, the H-manager can more effectively separate herself by withholding the forecast and pooling with L rather than forecasting and pooling with M.

Recall that we refer to the interval under which V_M/V_H satisfies condition (i) as the *valid region of* V_M/V_H. To better understand condition (i), we examine the following numerical example. Figure 5 shows how the valid region of V_M/V_H supporting the counter-signaling equilibrium changes with b_L. The green shaded area of Figure 5 represents the valid region of V_M/V_H. We see that, as b_L increases, the region of V_M/V_H expands. This is indirectly due to the fact that $\rho = 1$, which thus provides an additional lever to prevent deviation by the L-manager. As b_L increases, it becomes less likely that the L-manager can successfully mimic a higher ability manager, thus relaxing the region of V_M/V_H for which the equilibrium holds.

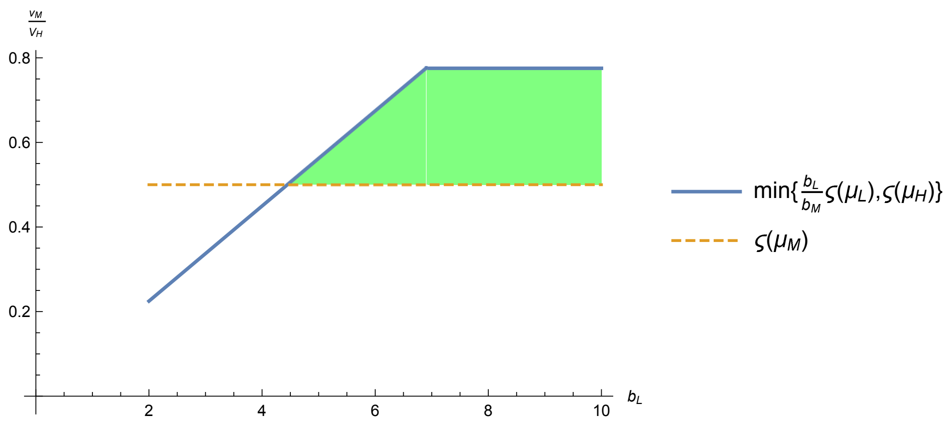

Figure 5: Changes in the valid region of V_M/V_H, captured by the green region, with changes in b_L, where $e|\theta \sim N(\mu_\theta, 1), \mu_H = 2, \mu_M = 1, \mu_L = 0, b_H = 1, b_M = 2, p_L = p_H = 1/4$, and $p_M = 1/2$.

5 Empirical Implications

The results of the model help to explain several empirical and anecdotal regularities and offer some new empirical predictions. First, the results help to explain the anomalous variation in the performance of firms that provide guidance. As mentioned previously, numerous well-performing firms choose not to issue guidance, and rather pool with many firms that have performed poorly. The model offers a theoretical explanation for such occurrences, in that high-type firms can better distinguish themselves from firms of intermediate ability by withholding guidance, even though this strategy is mimicked by low ability managers. Consequently, most of the earnings guidance that arises in equilibrium is issued by managers of intermediate ability.

Building from this, our results predict a greater proportion or a higher frequency of forecasts issued by firms with relatively intermediate performance. In other words, there should be an inverted U-shape in the probability of forecasting with respect to earnings, whereby the likelihood is relatively lower for low and high performing firms, and comparatively higher for firms with intermediate performance. Although we do not intend to provide any empirical evidence for our predictions, descriptive evidence provided in the Introduction supports this prediction. Figure 1 displays the likelihood of guidance issuance for different levels of scaled earnings (i.e., ROA), and shows the predicted inverted U-shape.

Moreover, we find that counter-signaling relies on a sufficiently high ex ante distribution of poorly performing firms (i.e., high p_L). This implies that the proportion of non-forecasting firms should fall more heavily on the relatively worse performing firms, and that the average price reaction to non-issuance of guidance should be *negative*.[17] A few studies in the empirical literature have found some evidence consistent with poor-performing firms forecasting less often. Ajinkya et al. (2005) documents that "loss firms," or firms with negative or zero earnings, forecast less frequently than non-loss firms. Similarly, using proxies for managerial ability, Baik et al. (2011) find that low-ability managers tend to forecast less often. This is consistent with the prediction that intermediate-performing firms are the most frequent forecasters, however, the results of these studies do not test separately the difference in forecasting frequency among intermediate and high performing firms or managers. Among this partitioned sample, our results predict a *negative* relation between forecasting frequency and performance.

Additionally, several studies have found evidence consistent with non-issuing firms receiving an average negative market reaction. This has been documented in Cheng et al. (2005), Houston et al. (2010), and Chen et al. (2011). We expect the price reaction for non-issuing firms to be negative *on average,* as the model predicts that the majority of non-issuing firms are low-performing in the counter signaling equilibrium. However, in a partitioned sample analysis which includes only medium and high-performing firms, we should expect the opposite result.

The results also provide predictions regarding characteristics of industries for which counter-signaling by high-performing firms is likely to be more prevalent. Proposition 1 shows that the level of managerial discretion over the decision to issue guidance (parameter ρ) must be sufficiently high in order for counter-signaling to be sustainable as an equilibrium. This implies that we are more likely to observe counter-signaling behavior in industries where managers have significant control over firm policies, or in firms/industries where the corporate governance structure or the board of directors is not as strong. Propositions 1 and 2 also show that there must be sufficient distance between managers' forecasting abilities across different types (parameter σ_θ or b_θ). This implies that we should expect more counter-

[17]Indeed, in Figure 1, we observe that the proportion of low-earnings firms that do not issue guidance is much larger than that of the high-earnings firms that follow the same strategy.

signaling behavior by high-type firms in industries where there is greater uncertainty over earnings, or among more complex firms where forecasting may be more difficult (e.g. Coles, Daniel, and Naveen, 2008). Equivalently, we should expect greater forecasting frequency among intermediate-performing firms in these industries.

A central condition for counter-signaling to hold is that the ratio V_M/V_H falls within a valid interval, as shown in Propositions 1 and 2. As we see from the comparative analysis in Section 4, the lower bound is generally more inelastic than the upper bound. This implies that counter-signaling is more likely to occur in industries where there is relatively less dispersion in performance between high and intermediate firms. For example, industries in which intermediate performers are relatively stronger are also more likely to exhibit the counter-signaling forecasting pattern.

The empirical motivation for our main assumption is provided by Goodman et al. (2013), who finds that managers who are better earnings forecasters also make more profitable investment decisions. The results of our model provide additional implications that build on Goodman et al. (2013). Specifically, the results imply that counter-signaling is more likely to occur in industries where the correlation between firm performance and managerial forecasting ability is stronger. For example, in industries where product development is rapidly changing, managers who are more adept at identifying and investing in profitable new technologies are likely to have stronger performance.

In summary, the frequency of forecasting by firms of intermediate performance should be greater (relative to low and high-performing firms) in industries where (*i*) managers have relatively greater control over firm policies; (*ii*) there is higher earnings uncertainty; or (*iii*) firm performance is relatively less heterogeneous.

6 Extensions

In this section, we first return to the uniform specification to explore the case where the manager may not have discretion, i.e., $\rho < 1$, and verify that the conditions for existence and uniqueness are qualitatively similar to that of the normal case. We then extend the model to a continuous type space.

6.1 Uniformly Distributed Error with Partial Discretion

The uniform setting with $\rho < 1$ is a more constrained problem than the case of $\rho = 1$ because it does not allow for the possibility of unequivocal separation, as some managers forecast regardless of their type. A key distinction between the full and partial discretion cases is that, if $\rho < 1$, the market *always* factors in the earnings announcement when updating their beliefs of the manager's type. In contrast, in the counter-signaling equilibrium with $\rho = 1$, the market updates using the announcement e only if the manager withheld issuing guidance. Specifically, in the counter-signaling equilibrium, the market price for a firm with forecast error ε and earnings announcement e is given by:

$$P(e, \varepsilon) = \frac{p_M \frac{\mathbb{I}_{|\varepsilon| \leq b_M}}{b_M} g\left(e|M\right) V_M + (1-\rho)\, p_H \frac{\mathbb{I}_{|\varepsilon| \leq b_H}}{b_H} g\left(e|H\right) V_H}{(1-\rho)\left[p_L \frac{\mathbb{I}_{|\varepsilon| \leq b_L}}{b_L} g\left(e|L\right) + p_H \frac{\mathbb{I}_{|\varepsilon| \leq b_H}}{b_H} g\left(e|H\right)\right] + p_M \frac{\mathbb{I}_{|\varepsilon| \leq b_M}}{b_M} g\left(e|M\right)}.$$

The market price of a firm with earnings e and no issued guidance is given by (10).

As we expect the market to generally use all relevant public information, the condition $\rho < 1$ thus leads to some more realistic equilibrium properties which are absent in the full discretion case. We characterize the counter-signaling equilibrium in the following proposition:

Proposition 3. *Assume $\rho < 1$ and errors are uniformly distributed. When $\frac{b_L}{b_M}$, $\frac{b_H}{b_M}$ and ρ are sufficiently large, there exist $\underline{r}$ and $\overline{r}$ with $\underline{r} < \overline{r}$ such that a counter-signaling equilibrium uniquely exists if and only if $\frac{V_M}{V_H} \in (\underline{r}, \overline{r})$, where $\underline{r}$ and $\overline{r}$ are specified in Appendix A.9.*

We see that a counter-signaling equilibrium exists given certain conditions on the bounds of the error as well as on the firm value. The underlying intuition is as follows. If b_L/b_M is large enough, an L-manager with discretion that forecasts is much more likely to be identified as being of an L type due to a large forecast error. Thus, L-managers with discretion prefer to withhold guidance. On the other hand, when b_H/b_M is sufficiently high, issuing guidance is not as effective for H-managers with discretion to differentiate themselves from M-managers, since their forecasting abilities are very close. Moreover, if ρ is large enough, by providing a forecast, M-managers with discretion are less likely to be misidentified as L types without discretion, and thus choose to issue guidance in equilibrium.

6.2 Continuous Types

In the baseline setting, we assume that manager type is discrete. While the baseline setting captures the main insights of the model, in this section we extend our setting to a continuous type space, i.e., $\theta \in \Theta \equiv [\theta_{\min}, \theta_{\max}]$. We denote the density function of θ by $f(\theta)$ over the support Θ. We continue to use the same notation as in the baseline setting. We assume that both $g(e|\theta)$ and $h(\varepsilon|\theta)$ are continuous in $\theta \in [\theta_{\min}, \theta_{\max}]$. Moreover, the family of densities $g(e|\theta)$ satisfy the monotone likelihood ratio property, i.e., $\forall \theta_1 > \theta_0$, $\forall e_1 > e_0$, $\frac{g(e_1|\theta_1)}{g(e_1|\theta_0)} \geq \frac{g(e_0|\theta_1)}{g(e_0|\theta_0)}$. We continue to assume that all earnings distributions have the same support regardless of the manager's ability. Moreover, we assume that the distribution of the forecast error is symmetric with mean 0, and the forecast of a manager with higher talent is more precise and therefore less dispersed, i.e., $\forall \theta_1 > \theta_0$, $\forall \varepsilon_1 < \varepsilon_0 < 0$, $\frac{h(\varepsilon_1|\theta_0)}{h(\varepsilon_1|\theta_1)} \geq \frac{h(\varepsilon_0|\theta_0)}{h(\varepsilon_0|\theta_1)}$. We analogously define counter-signaling in this continuous setting as follows:

Definition 3. A *counter-signaling* equilibrium is an equilibrium in which, among managers with discretion, only managers of intermediate ability choose to issue guidance, while sufficiently low or sufficiently high ability managers with discretion only report the realized earnings. Formally, $\exists \theta_1, \theta_2 \in [\theta_{\min}, \theta_{\max}]$, s.t., $\theta_1 < \theta_2$ and $\Theta_F^* = [\theta_1, \theta_2]$.

While the full analytical analysis of this setting becomes intractable, we provide properties of the equilibria as well as a numerical analysis. A counter-signaling equilibrium is characterized by two thresholds, $\theta_1 > \theta_{\min}$, $\theta_2 < \theta_{\max}$, and $\theta_1 < \theta_2$, such that only managers with $\theta \in [\theta_1, \theta_2]$ issue guidance. We first show that a single threshold equilibrium does not exist.

Proposition 4. *Assume $\rho \in (0, 1]$. There does not exist an equilibrium with a single threshold, θ^*, such that all managers of type $\theta > \theta^*$ with discretion issue guidance, and all managers of type $\theta \leq \theta^*$ with discretion withhold guidance, or vice versa. Formally, $\nexists \theta^* \in (\theta_{\min}, \theta_{\max})$, s.t., $\Theta_F^* = [\theta_{\min}, \theta^*]$ or $\Theta_F^* = [\theta^*, \theta_{\max}]$.*

Proposition 4 implies that any equilibrium without full pooling must have at least two thresholds. This means that there will be a region (or multiple regions) of manager types which provide guidance in equilibrium, while the others withhold. Intuitively, this occurs

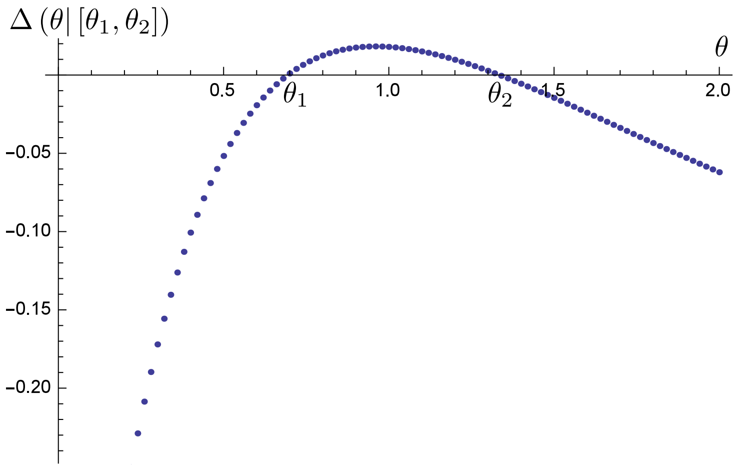

Figure 6: Changes in the expected payoff difference from issuing guidance and keeping quiet for various levels of managerial ability. Managers whose types admit a non-negative difference choose to issue guidance.

due to the incentive of mimicry by intermediate types. For example, a type just below θ^* is pooled with the lower end of the support, and thus can improve her price by issuing guidance and mimicking a type above θ^*.

We now numerically examine whether a counter-signaling equilibrium exists. In a counter-signaling equilibrium with $\Theta_F = [\theta_1, \theta_2]$, a manager of type $\theta \in \Theta_F$ with discretion prefers to issue guidance, i.e., $\forall \theta \in [\theta_1, \theta_2]$, $\Delta(\theta|\, [\theta_1, \theta_2]) \geq 0$. Likewise, a manager with discretion of type $\theta \notin \Theta_F$ prefers to keep quiet, i.e., $\forall \theta \in [\theta_{\min}, \theta_1] \cup [\theta_2, \theta_{\max}]$, $\Delta(\theta|\, [\theta_1, \theta_2]) < 0$. Therefore, the equilibrium thresholds θ_1 and θ_2 are jointly determined by:

$$\Delta(\theta_1|\, [\theta_1, \theta_2]) = 0,$$
$$\Delta(\theta_2|\, [\theta_1, \theta_2]) = 0.$$

We provide a numerical example to verify that a counter-signaling equilibrium exists under this continuous-type space. We specify the following distributional values: the manager type is uniformly distributed between $\theta_{\min}$ and $\theta_{\max}$, i.e., $f(\theta) = \frac{1}{\theta_{\max} - \theta_{\min}}$; conditional earnings are normally distributed, $e|\theta \sim N(\mu_\theta, 1)$, where the mean is given as $\mu_\theta = V_\theta = \theta$. The forecast error is also normally distributed, $\varepsilon|\theta \sim N(0, \sigma_\theta)$, with variance

$\sigma_\theta = \frac{1}{\theta}$. We set $\theta_{\min} = 0.1, \theta_{\max} = 2$, and $\rho = 2/3$. This specification admits a numerical solution where there are exactly two threshold levels for which manager types between these thresholds issue guidance. These are given by $\theta_1 = 0.6936$ and $\theta_2 = 1.3291$ in the counter-signaling equilibrium. Figure 6 shows how the expected payoff difference between forecasting and keeping quiet changes with managerial ability. We see that $\forall \theta \in [\theta_1, \theta_2]$, $\Delta(\theta | [\theta_1, \theta_2]) \geq 0$ and $\forall \theta \in [\theta_{\min}, \theta_1] \cup [\theta_2, \theta_{\max}]$, $\Delta(\theta | [\theta_1, \theta_2]) < 0$, which satisfies the equilibrium conditions.

7 Concluding remarks

This paper helps to explain the observed regularity that both highly successful as well as poorly performing firms tend to withhold issuing earnings guidance. We develop a model where the firm's earnings and the accuracy of the manager's signal are increasing in her ability. This leads both high and low type firms to refrain from issuing a forecast. Low type firms keep quiet as more information provided to the market limits their ability to pool, while high type firms withhold guidance to more clearly distinguish themselves from the intermediate-performing firms. That is, high type firms *counter-signal* in equilibrium to separate themselves. Our general analysis shows that, under partial discretion of the guidance choice, counter-signaling is the only partial separating equilibrium that is feasible. We provide specific conditions of the existence and uniqueness of the counter-signaling equilibrium using two commonly used distributions: the uniform and normal. The conditions we find are qualitatively similar under both cases.

The results provide a rich set of empirical predictions that can be investigated in future work. Our central characterization of counter-signaling implies that we should expect an inverted U-shape regarding the frequency of managerial forecasting with respect to performance, which matches a preliminary examination of the data (see Figure 1). While we have provided initial evidence of this stylized fact, future work can build on this and provide a comprehensive empirical analysis of this counter-signaling pattern. For example, the results predict that counter-signaling is more likely to emerge or is more prevalent in industries where managers have more control over firm policies, there is greater uncertainty over earnings, or firm performance is less heterogeneous.

References

Acharya, V. V., P. DeMarzo, and I. Kremer (2011): "Endogenous information flows and the clustering of announcements," *The American Economic Review*, 101, 2955–2979.

Aghamolla, C. and T. Hashimoto (2019): "Managerial forecast manipulation and distortive investment," *Working paper*.

Ajinkya, B., S. Bhojraj, and P. Sengupta (2005): "The association between outside directors, institutional investors and the properties of management earnings forecasts," *Journal of Accounting Research*, 43, 343–376.

Araujo, A., D. Gottlieb, and H. Moreira (2007): "A model of mixed signals with applications to countersignalling," *The RAND Journal of Economics*, 38, 1020–1043.

Arya, A. and B. Mittendorf (2005): "Using disclosure to influence herd behavior and alter competition," *Journal of Accounting and Economics*, 40, 231–246.

——— (2016): "On the synergy between disclosure and investment beauty contests," *Journal of Accounting and Economics*, 61, 255–273.

Arya, A., B. Mittendorf, and A. Sudbury (2015): "Forward contracting and incentives for disclosure," *Review of Accounting Studies*, 20, 1093–1121.

Baik, B., D. B. Farber, and S. S. Lee (2011): "CEO ability and management earnings forecasts," *Contemporary Accounting Research*, 28, 1645–1668.

Bamber, L. S., J. Jiang, and I. Y. Wang (2010): "Whats my style? The influence of top managers on voluntary corporate financial disclosure," *The accounting review*, 85, 1131–1162.

Beyer, A. (2009): "Capital market prices, management forecasts, and earnings management," *The Accounting Review*, 84, 1713–1747.

Beyer, A. and R. A. Dye (2012): "Reputation management and the disclosure of earnings forecasts," *Review of Accounting Studies*, 17, 877–912.

Brochet, F., L. Faurel, and S. McVay (2011): "Manager-specific effects on earnings guidance: An analysis of top executive turnovers," *Journal of Accounting Research*, 49, 1123–1162.

Chen, S., D. Matsumoto, and S. Rajgopal (2011): "Is silence golden? An empirical analysis of firms that stop giving quarterly earnings guidance," *Journal of Accounting and Economics*, 51, 134–150.

Cheng, M., K. Subramanyam, and Y. Zhang (2005): "Earnings guidance and managerial myopia," *Working paper*.

Coles, J. L., N. D. Daniel, and L. Naveen (2008): "Boards: Does one size fit all?" *Journal of Financial Economics*, 87, 329–356.

Diamond, D. W. and R. E. Verrecchia (1991): "Disclosure, liquidity, and the cost of capital," *The journal of Finance*, 46, 1325–1359.

Edmans, A., M. S. Heinle, and C. Huang (2016): "The real costs of financial efficiency when some information is soft," *Review of Finance*, 20, 2151–2182.

Feltovich, N., R. Harbaugh, and T. To (2002): "Too cool for school? Signalling and countersignalling," *The Rand Journal of Economics*, 33, 630.

Gervais, S. and G. Strobl (2015): "Transparency and Talent Allocation in Money Management," *Working paper.*

Gibbins, M., A. Richardson, and J. Waterhouse (1990): "The management of corporate financial disclosure: opportunism, ritualism, policies, and processes," *Journal of accounting research*, 28, 121–143.

Goodman, T. H., M. Neamtiu, N. Shroff, and H. D. White (2013): "Management forecast quality and capital investment decisions," *The Accounting Review*, 89, 331–365.

Göx, R. F. and A. Wagenhofer (2009): "Optimal impairment rules," *Journal of Accounting and Economics*, 48, 2–16.

Graham, J. R., C. R. Harvey, and S. Rajgopal (2005): "The economic implications of corporate financial reporting," *Journal of Accounting and Economics*, 40, 3–73.

Grossman, S. J. (1981): "The informational role of warranties and private disclosure about product quality," *The Journal of Law and Economics*, 24, 461–483.

Grossman, S. J. and O. D. Hart (1980): "Disclosure laws and takeover bids," *The Journal of Finance*, 35, 323–334.

Heinle, M. S. and R. E. Verrecchia (2015): "Bias and the Commitment to Disclosure," *Management Science*, 62, 2859–2870.

Houston, J. F., B. Lev, and J. W. Tucker (2010): "To guide or not to guide? causes and consequences of stopping quarterly earnings guidance," *Contemporary Accounting Research*, 27, 143–185.

Jehiel, P. (2015): "On transparency in organizations," *The Review of Economic Studies*, 82, 736–761.

Kamenica, E. and M. Gentzkow (2011): "Bayesian persuasion," *American Economic Review*, 101, 2590–2615.

Li, H. and X. Shi (2017): "Discriminatory information disclosure," *American Economic Review*, 107, 3363–85.

Marinovic, I. and S. S. Sridhar (2015): "Discretionary disclosures using a certifier," *Journal of Accounting and Economics*, 59, 25–40.

Mccormack, K. (2007): "Giving up on Guidance," *Bloomberg Businessweek*, May 14, 2007.

Milgrom, P. R. (1981): "Good news and bad news: Representation theorems and applications," *The Bell Journal of Economics*, 380–391.

Osterland, A. (2004): "The Best CFOs in America," *Institutional Investor*, February 14, 2004.

Ottaviani, M. and P. N. Sørensen (2006): "Reputational cheap talk," *The Rand Journal of Economics*, 37, 155–175.

Page, L. and S. Brin (2004): "'An Owners Manual' for Googles Shareholders," *2004 Founders IPO Letter*.

Ramakrishnan, R. T. and X. Wen (2016): "Meeting company-issued guidance and management guidance strategy," *Journal of Law, Finance, and Accounting*, 1, 319–359.

Rayo, L. and I. Segal (2010): "Optimal information disclosure," *Journal of political Economy*, 118, 949–987.

Teoh, S. H. and C. Y. Hwang (1991): "Nondisclosure and adverse disclosure as signals of firm value," *Review of Financial Studies*, 4, 283–313.

Trueman, B. (1986): "Why do managers voluntarily release earnings forecasts?" *Journal of Accounting and Economics*, 8, 53–71.

A Appendix: Proofs

A.1 Proof of Lemma 1

First, $\forall e_1 > e_0, \theta_1 > \theta_0$, we have

$$\frac{\gamma\left(\theta_1|e_1,\varepsilon;\hat{\Theta}_F\right)}{\gamma\left(\theta_1|e_0,\varepsilon;\hat{\Theta}_F\right)} = \frac{1_{\theta_1\in\Theta\backslash\hat{\Theta}_F}\left(1-\rho\right)f\left(\theta_1\right)g\left(e_1|\theta_1\right)h\left(\varepsilon|\theta_1\right)+1_{\theta_1\in\hat{\Theta}_F}f\left(\theta_1\right)g\left(e_1|\theta_1\right)h\left(\varepsilon|\theta_1\right)}{1_{\theta_1\in\Theta\backslash\hat{\Theta}_F}\left(1-\rho\right)f\left(\theta_1\right)g\left(e_0|\theta_1\right)h\left(\varepsilon|\theta_1\right)+1_{\theta_1\in\hat{\Theta}_F}f\left(\theta_1\right)g\left(e_0|\theta_1\right)h\left(\varepsilon|\theta_1\right)}$$
$$= \frac{g\left(e_1|\theta_1\right)}{g\left(e_0|\theta_1\right)}.$$

Replacing θ_1 with θ_0, we have $\frac{\gamma\left(\theta_0|e_1,\varepsilon;\hat{\Theta}_F\right)}{\gamma\left(\theta_0|e_0,\varepsilon;\hat{\Theta}_F\right)} = \frac{g\left(e_1|\theta_0\right)}{g\left(e_0|\theta_0\right)}$.

By the monotone likelihood ratio property, we have $\frac{g\left(e_1|\theta_1\right)}{g\left(e_1|\theta_0\right)} \geq \frac{g\left(e_0|\theta_1\right)}{g\left(e_0|\theta_0\right)}$. Therefore, we can get:

$$\frac{g\left(e_1|\theta_1\right)}{g\left(e_1|\theta_0\right)} \geq \frac{g\left(e_0|\theta_1\right)}{g\left(e_0|\theta_0\right)} \Rightarrow \frac{g\left(e_1|\theta_1\right)}{g\left(e_0|\theta_1\right)} \geq \frac{g\left(e_1|\theta_1\right)}{g\left(e_0|\theta_1\right)} \Rightarrow \frac{\gamma\left(\theta_1|e_1,\varepsilon;\hat{\Theta}_F\right)}{\gamma\left(\theta_1|e_0,\varepsilon;\hat{\Theta}_F\right)} \geq \frac{\gamma\left(\theta_0|e_1,\varepsilon;\hat{\Theta}_F\right)}{\gamma\left(\theta_0|e_0,\varepsilon;\hat{\Theta}_F\right)},$$

which implies that $\gamma\left(\theta|e,\varepsilon\right)$ also satisfies the monotone likelihood ratio property, and thus exhibits first-order stochastic dominance in θ. Moreover, since V_θ is increasing in θ (i.e., $V_H \geq V_M \geq V_L$), we obtain

$$\sum_{\theta\in\Theta}\gamma\left(\theta_1|e_1,\varepsilon;\hat{\Theta}_F\right)V_\theta \geq \sum_{\theta\in\Theta}\gamma\left(\theta_1|e_0,\varepsilon;\hat{\Theta}_F\right)V_\theta,$$

which implies $P\left(e,\varepsilon\right)$ is increasing in e.

We can similarly prove that $P\left(e,\varepsilon\right)$ is increasing in ε if $\varepsilon < 0$ and decreasing in ε if $\varepsilon > 0$, and that $P\left(e,\emptyset\right)$ is increasing in e.

A.2 Proof of Theorem 1

Since $\Theta_F^* \subseteq \Theta$, we have $\Theta_F^* \in 2^\Theta$. First, we show that $\forall\rho \in (0,1]$, $\Theta_F^* \in \left\{\emptyset,\Theta,\{H\},\{M\}\right\}$ in any equilibrium. It is equivalent to show in any equilibrium that $\forall\rho \in (0,1]$, we have $\Theta_F^* \notin \left\{\{L\},\{L,M\},\{M,H\},\{L,H\}\right\}$.

(i) $\Theta_F^* \neq \{L\}$: Assume that the equilibrium is $\Theta_F^* = \{L\}$. Since only M and H-managers choose non-forecast, the equilibrium market price of a firm with earning e without earnings forecast is:

$$P\left(e,\emptyset\right) = E\left[v|e,\emptyset\right] = \frac{p_M g\left(e|M\right)V_M + p_H g\left(e|H\right)V_H}{p_M g\left(e|M\right) + p_H g\left(e|H\right)} > V_M.$$

On the other hand, since both L-managers with discretion and all managers without discretion choose forecast, the equilibrium market price of a firm with earnings report e and earnings forecast $y = e + \varepsilon$ is:

$$P\left(e,\varepsilon\right) = \frac{\left(1-\rho\right)\left[p_M g\left(e|M\right)h\left(\varepsilon|M\right)V_M + p_H g\left(e|H\right)h\left(\varepsilon|H\right)V_H\right]}{p_L g\left(e|L\right)h\left(\varepsilon|L\right) + \left(1-\rho\right)\left[p_M g\left(e|M\right)h\left(\varepsilon|M\right) + p_H g\left(e|H\right)h\left(\varepsilon|H\right)\right]}.$$

Note that since the forecast by managers of higher ability is more precise, we can obtain

$$
\begin{aligned}
LHS \equiv & \int_{-\infty}^{\infty} \frac{h\left(\varepsilon|L\right)\left[p_M g\left(e|M\right) h\left(\varepsilon|M\right) V_M + p_H g\left(e|H\right) h\left(\varepsilon|H\right) V_H\right]}{\left[p_M g\left(e|M\right) h\left(\varepsilon|M\right) + p_H g\left(e|H\right) h\left(\varepsilon|H\right)\right]} d\varepsilon \\
< & \frac{p_M g\left(e|M\right) V_M + p_H g\left(e|H\right) V_H}{p_M g\left(e|M\right) + p_H g\left(e|H\right)} \equiv RHS.
\end{aligned}
$$

This inequality is proved as following:

$$
\begin{aligned}
& LHS - RHS \\
= & \int_{-\infty}^{\infty} h\left(\varepsilon|L\right) \frac{p_H g\left(e|H\right) p_M g\left(e|M\right)\left[h\left(\varepsilon|H\right) - h\left(\varepsilon|M\right)\right]\left(V_H - V_M\right)}{\left[p_M g\left(e|M\right) h\left(\varepsilon|M\right) + p_H g\left(e|H\right) h\left(\varepsilon|H\right)\right]\left[p_M g\left(e|M\right) + p_H g\left(e|H\right)\right]} d\varepsilon \\
= & \frac{p_H g\left(e|H\right) p_M g\left(e|M\right)\left(V_H - V_M\right)}{\left[p_M g\left(e|M\right) + p_H g\left(e|H\right)\right]} \int_{-\infty}^{\infty} h\left(\varepsilon|L\right) \frac{\left[h\left(\varepsilon|H\right) - h\left(\varepsilon|M\right)\right]}{\left[p_M g\left(e|M\right) h\left(\varepsilon|M\right) + p_H g\left(e|H\right) h\left(\varepsilon|H\right)\right]} d\varepsilon,
\end{aligned}
$$

so, $LHS - RHS < 0$ is equivalent to:

$$
\int_{-\infty}^{\infty} h\left(\varepsilon|L\right) \frac{\left[h\left(\varepsilon|H\right) - h\left(\varepsilon|M\right)\right]}{\left[p_M g\left(e|M\right) h\left(\varepsilon|M\right) + p_H g\left(e|H\right) h\left(\varepsilon|H\right)\right]} d\varepsilon < 0.
$$

Since $h(\varepsilon|\theta)$ is symmetric around zero, the above inequality can be further simplified as:

$$
\begin{aligned}
& \int_{-\infty}^{0} \frac{1}{\left[p_M g\left(e|M\right) \frac{h\left(\varepsilon|M\right)}{h\left(\varepsilon|L\right)} + p_H g\left(e|H\right) \frac{h\left(\varepsilon|H\right)}{h\left(\varepsilon|L\right)}\right]} dH\left(\varepsilon|H\right) \\
< & \int_{-\infty}^{0} \frac{1}{\left[p_M g\left(e|M\right) \frac{h\left(\varepsilon|M\right)}{h\left(\varepsilon|L\right)} + p_H g\left(e|H\right) \frac{h\left(\varepsilon|H\right)}{h\left(\varepsilon|L\right)}\right]} dH\left(\varepsilon|M\right).
\end{aligned}
$$

Both $\frac{h\left(\varepsilon|M\right)}{h\left(\varepsilon|L\right)}$ and $\frac{h\left(\varepsilon|H\right)}{h\left(\varepsilon|L\right)}$ are increasing functions of ε when $\varepsilon < 0$, which implies $\frac{1}{\left[p_M g\left(e|M\right) \frac{h\left(\varepsilon|M\right)}{h\left(\varepsilon|L\right)} + p_H g\left(e|H\right) \frac{h\left(\varepsilon|H\right)}{h\left(\varepsilon|L\right)}\right]}$ is a decreasing function of ε when $\varepsilon < 0$. Meanwhile, by the property of monotone likelihood ratio, we have $\left\{\varepsilon|H, \varepsilon < 0\right\}$ first order dominates $\left\{\varepsilon|M, \varepsilon < 0\right\}$. Therefore, the inequality above holds. The intuition is that providing a forecast makes the low type easier being identified as low type. Using this result, we can obtain

$$
\begin{aligned}
&E\left[P\left(e,\varepsilon\right)|L\right] \\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{h\left(\varepsilon|L\right)g\left(e|L\right)\left(1-\rho\right)\left[p_{M}g\left(e|M\right)h\left(\varepsilon|M\right)V_{M}+p_{H}g\left(e|H\right)h\left(\varepsilon|H\right)V_{H}\right]}{p_{L}g\left(e|L\right)h\left(\varepsilon|L\right)+\left(1-\rho\right)\left[p_{M}g\left(e|M\right)h\left(\varepsilon|M\right)+p_{H}g\left(e|H\right)h\left(\varepsilon|H\right)\right]}d\varepsilon de \\
&<\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{h\left(\varepsilon|L\right)g\left(e|L\right)\left[p_{M}g\left(e|M\right)h\left(\varepsilon|M\right)V_{M}+p_{H}g\left(e|H\right)h\left(\varepsilon|H\right)V_{H}\right]}{\left[p_{M}g\left(e|M\right)h\left(\varepsilon|M\right)+p_{H}g\left(e|H\right)h\left(\varepsilon|H\right)\right]}d\varepsilon de \\
&<\int_{-\infty}^{\infty}\frac{g\left(e|L\right)\left[p_{M}g\left(e|M\right)V_{M}+p_{H}g\left(e|H\right)V_{H}\right]}{p_{M}g\left(e|M\right)+p_{H}g\left(e|H\right)}de \\
&=E\left[P\left(e,\emptyset\right)|L\right],
\end{aligned}
$$

which implies that by withholding earnings forecast, L-managers can improve the price of their firm. Therefore, L-managers have incentives to deviate from forecasting to non-forecasting. Thus, it is impossible that only L-managers choose earnings forecast in an equilibrium, i.e., $\Theta_F^* \neq \{L\}$.

(ii) $\Theta_F^* \neq \{L, M\}$: Since only H-managers choose non-forecast, the equilibrium market price of a firm without earnings forecast is:

$$P\left(e,\emptyset\right) = V_H.$$

On the other hand, since both L and M-managers with discretion and all managers without discretion choose to forecast, the equilibrium market price of a firm with an earnings forecast is strictly lower than V_H, i.e., $P\left(e,\varepsilon\right) < V_H$. By not forecasting, L and M-managers can increase the price of their firm. Therefore, L and M-managers have incentives to deviate from forecasting to not forecasting. Thus, it is impossible that only L and M-managers choose to forecast in equilibrium, i.e., $\Theta_F^* \neq \{L, M\}$.

(iii) $\Theta_F^* \neq \{M, H\}$: Since only L-managers choose non-forecast, the equilibrium market price of a firm without earnings forecast is:

$$P\left(e,\emptyset\right) = V_L = 0.$$

On the other hand, since both M and H-managers with discretion and all managers without discretion forecast, the equilibrium market price of a firm with an earnings forecast is strictly greater than V_L, i.e., $P\left(e,\varepsilon\right) > V_L$. By forecasting, L-managers may successfully pool with M or H-managers and thus improve the price of their firm. Therefore, L-managers have incentives to deviate from forecasting to non-forecasting. Thus, it is impossible that only M and H-managers choose to forecast in an equilibrium, i.e., $\Theta_F^* \neq \{M, H\}$.

(iv) $\Theta_F^* \neq \{L, H\}$: Since only M-managers choose non-forecast, the equilibrium market price of a firm without an earnings forecast is:

$$P\left(e,\emptyset\right) = V_M.$$

On the other hand, since both L and H-managers with discretion and all managers without discretion forecast, the equilibrium market price of a firm with an earnings report e and an earnings forecast $y = e + \varepsilon$ is:

$$P(e,\varepsilon) = \frac{(1-\rho)\, p_M g\left(e|M\right) h\left(\varepsilon|M\right) V_M + p_H g\left(e|H\right) h\left(\varepsilon|H\right) V_H}{p_L g\left(e|L\right) h\left(\varepsilon|L\right) + (1-\rho)\, p_M g\left(e|M\right) h\left(\varepsilon|M\right) + p_H g\left(e|H\right) h\left(\varepsilon|H\right)},$$

which is between V_L and V_H. Note that:

$$\begin{aligned} P(e,\varepsilon) = V_M &- \frac{p_L g\left(e|L\right) V_M}{p_L g\left(e|L\right) + (1-\rho)\, p_M g\left(e|M\right) \frac{h(\varepsilon|M)}{h(\varepsilon|L)} + p_H g\left(e|H\right) \frac{h(\varepsilon|H)}{h(\varepsilon|L)}} \\ &+ \frac{p_H g\left(e|H\right)\left(V_H - V_M\right)}{p_L g\left(e|L\right) \frac{h(\varepsilon|L)}{h(\varepsilon|H)} + (1-\rho)\, p_M g\left(e|M\right) \frac{h(\varepsilon|M)}{h(\varepsilon|H)} + p_H g\left(e|H\right)}, \end{aligned}$$

which is increasing in ε for $\varepsilon < 0$ since both $\frac{h(\varepsilon|M)}{h(\varepsilon|L)}$ and $\frac{h(\varepsilon|H)}{h(\varepsilon|L)}$ are increasing in $\varepsilon < 0$, and both $\frac{h(\varepsilon|L)}{h(\varepsilon|H)}$ and $\frac{h(\varepsilon|M)}{h(\varepsilon|H)}$ are decreasing in $\varepsilon < 0$. As it is an equilibrium, L-managers does not have an incentive to deviate, which implies the expected payoff is higher for L-managers by choosing to forecast rather than not to forecast, that is,

$$\Delta\left(L|\Theta_F = \{L,H\}\right) \geq 0 \Leftrightarrow E\left[P(e,\varepsilon)\,|L\right] \geq E\left[P\left(e,\emptyset\right)|L\right] = V_M.$$

Moreover, since M-managers can forecast more precisely than L-managers, an M-manager has a higher chance to successfully pool with H-managers by forecasting, that is,

$$\begin{aligned} & E\left[P(e,\varepsilon)\,|M\right] > E\left[P(e,\varepsilon)\,|L\right] \\ \Leftrightarrow & \int_{-\infty}^{\infty} P(e,\varepsilon)\, h\left(\varepsilon|M\right) d\varepsilon > \int_{-\infty}^{\infty} p(e,\varepsilon)\, h\left(\varepsilon|L\right) d\varepsilon \\ \Leftrightarrow & \int_{-\infty}^{0} P(e,\varepsilon)\, h\left(\varepsilon|M\right) d\varepsilon > \int_{-\infty}^{0} P(e,\varepsilon)\, h\left(\varepsilon|L\right) d\varepsilon, \end{aligned}$$

which is true since $P(e,\varepsilon)$ is increasing in ε and $\frac{h(\varepsilon|M)}{h(\varepsilon|L)}$ is increasing in ε, when $\varepsilon < 0$. Therefore, we have $E\left[P(e,\varepsilon)\,|M\right] > V_M = E\left[p\left(e,\emptyset\right)|M\right]$. By forecasting, M-managers can increase the price of their firm. Therefore, M-managers have incentives to deviate from non-forecasting to forecasting. Thus, it is impossible that only L and H-managers choose to forecast in equilibrium, i.e., $\Theta_F^* \neq \{L,H\}$.

Thus, $\forall \rho \in (0,1]$, $\Theta_F^* \in \left\{\emptyset, \Theta, \{H\}, \{M\}\right\}$.

Now we show that $\Theta_F^* \notin \left\{\emptyset, \{H\}\right\}$, $\forall \rho \in (0,1)$.

(i) $\Theta_F^* \neq \emptyset$: Since all managers with discretion choose non-forecast, the equilibrium market price of a firm without earnings forecast is:

$$P\left(e,\emptyset\right) = E\left[v|e,\emptyset\right] = \frac{p_M g\left(e|M\right) V_M + p_H g\left(e|H\right) V_H}{p_L g\left(e|L\right) + p_M g\left(e|M\right) + p_H g\left(e|H\right)}.$$

On the other hand, with $\rho < 1$, there are managers without discretion providing forecast, so the equilibrium market price of a firm with earnings report e and earnings forecast $y = e + \varepsilon$ is:

$$P(e, \varepsilon) = E[v|e, \varepsilon] = \frac{p_M g(e|M) h(\varepsilon|M) V_M + p_H g(e|H) h(\varepsilon|H) V_H}{p_L g(e|L) h(\varepsilon|L) + p_M g(e|M) h(\varepsilon|M) + p_H g(e|H) h(\varepsilon|H)}.$$

Intuitively, H-managers provide more precise forecast than L and M-managers, so H-managers have a higher chance to differentiate themselves from L or M-managers by providing forecast. Mathematically, we want to prove that:

$$E\left[P(e, \varepsilon)|H\right] > E\left[P\left(e, \emptyset\right)|H\right],$$

which is equivalent to

$$\begin{aligned}
& E\left[P(e, \varepsilon)|H\right] - E\left[P\left(e, \emptyset\right)|H\right] \\
= & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} P(e, \varepsilon) h(\varepsilon|H) d\varepsilon - P\left(e, \emptyset\right)\right] g(e|H) de \\
= & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \left[p(e, \varepsilon) - P\left(e, \emptyset\right)\right] h(\varepsilon|H) d\varepsilon\right] g(e|H) de > 0.
\end{aligned}$$

Observing that

$$\begin{aligned}
& P(e, \varepsilon) - P\left(e, \emptyset\right) \\
= & \frac{V_M \left[h(\varepsilon|M) - h(\varepsilon|L)\right]}{\left[h(\varepsilon|L) + \frac{p_M g(e|M)}{p_L g(e|L)} h(\varepsilon|M) + \frac{p_H g(e|H)}{p_L g(e|L)} h(\varepsilon|H)\right]\left[\frac{p_L g(e|L)}{p_M g(e|M)} + 1 + \frac{p_H g(e|H)}{p_M g(e|M)}\right]} \\
+ & \frac{V_H \left[h(\varepsilon|H) - h(\varepsilon|L)\right]}{\left[h(\varepsilon|L) + \frac{p_M g(e|M)}{p_L g(e|L)} h(\varepsilon|M) + \frac{p_H g(e|H)}{p_L g(e|L)} h(\varepsilon|H)\right]\left[\frac{p_L g(e|L)}{p_H g(e|H)} + \frac{p_M g(e|M)}{p_H g(e|H)} + 1\right]} \\
+ & \frac{(V_H - V_M)\left[h(\varepsilon|H) - h(\varepsilon|M)\right]}{\left[\frac{p_L g(e|L)}{p_H g(e|H)} h(\varepsilon|L) + \frac{p_M g(e|M)}{p_H g(e|H)} h(\varepsilon|M) + h(\varepsilon|H)\right]\left[\frac{p_L g(e|L)}{p_M g(e|M)} + 1 + \frac{p_H g(e|H)}{p_M g(e|M)}\right]},
\end{aligned}$$

we can obtain

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\left[P\left(e,\varepsilon\right)-P\left(e,\emptyset\right)\right]h\left(\varepsilon|H\right)d\varepsilon\\
=&\int_{-\infty}^{\infty}\frac{V_M\left[h\left(\varepsilon|M\right)-h\left(\varepsilon|L\right)\right]}{\left[\frac{h(\varepsilon|L)}{h(\varepsilon|H)}+\frac{p_M g(e|M)}{p_L g(e|L)}\frac{h(\varepsilon|M)}{h(\varepsilon|H)}+\frac{p_H g(e|H)}{p_L g(e|L)}\right]\left[\frac{p_L g(e|L)}{p_M g(e|M)}+1+\frac{p_H g(e|H)}{p_M g(e|M)}\right]}d\varepsilon\\
&+\int_{-\infty}^{\infty}\frac{V_H\left[h\left(\varepsilon|H\right)-h\left(\varepsilon|L\right)\right]}{\left[\frac{h(\varepsilon|L)}{h(\varepsilon|H)}+\frac{p_M g(e|M)}{p_L g(e|L)}\frac{h(\varepsilon|M)}{h(\varepsilon|H)}+\frac{p_H g(e|H)}{p_L g(e|L)}\right]\left[\frac{p_L g(e|L)}{p_H g(e|H)}+\frac{p_M g(e|M)}{p_H g(e|H)}+1\right]}d\varepsilon\\
&+\int_{-\infty}^{\infty}\frac{\left(V_H-V_M\right)\left[h\left(\varepsilon|H\right)-h\left(\varepsilon|M\right)\right]}{\left[\frac{p_L g(e|L)}{p_H g(e|H)}\frac{h(\varepsilon|L)}{h(\varepsilon|H)}+\frac{p_M g(e|M)}{p_H g(e|H)}\frac{h(\varepsilon|M)}{h(\varepsilon|H)}+1\right]\left[\frac{p_L g(e|L)}{p_M g(e|M)}+1+\frac{p_H g(e|H)}{p_M g(e|M)}\right]}d\varepsilon\\
>&0,
\end{aligned}
$$

where the last inequality comes from the property of monotone likelihood ratio. Hence, we obtain

$$
\begin{aligned}
&E\left[P\left(e,\varepsilon\right)|H\right]-E\left[p\left(e,\emptyset\right)|H\right]\\
=&\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\left[p\left(e,\varepsilon\right)-P\left(e,\emptyset\right)\right]h\left(\varepsilon|H\right)d\varepsilon\right]g(e|H)de\\
>&\ 0.
\end{aligned}
$$

Therefore, H-managers have incentives to deviate from non-forecasting to forecasting. Thus, it is impossible that none managers with discretion choose to forecast in equilibrium, i.e., $\Theta_F^*\neq\emptyset$.

(ii) $\Theta_F^*\neq\{H\}$: Since only L and M-managers with discretion choose non-forecast, the equilibrium market price of a firm without earnings forecast is:

$$P\left(e,\emptyset\right)=\frac{p_M g\left(e|M\right)V_M}{p_L g\left(e|L\right)+p_M g\left(e|M\right)}.$$

On the other hand, since both H-managers with discretion and managers without discretion provide forecast, the equilibrium market price of a firm with earnings report e and earnings forecast $y=e+\varepsilon$ is:

$$
\begin{aligned}
P\left(e,\varepsilon\right)&=E\left[v|e,\varepsilon\right]\\
&=\frac{(1-\rho)\,p_M g\left(e|M\right)h\left(\varepsilon|M\right)V_M+p_H g\left(e|H\right)h\left(\varepsilon|H\right)V_H}{(1-\rho)\left[p_L g\left(e|L\right)h\left(\varepsilon|L\right)+p_M g\left(e|M\right)h\left(\varepsilon|M\right)\right]+p_H g\left(e|H\right)h\left(\varepsilon|H\right)}.
\end{aligned}
$$

Since M-managers forecast more precisely than L-managers, M-managers have a higher chance of differentiating themselves from L-managers by providing a forecast than by withholding it, which can be formalized as follows:

$$
\begin{aligned}
&E\left[P\left(e,\varepsilon\right)|M\right] \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{g\left(e|M\right)h\left(\varepsilon|M\right)\left[\left(1-\rho\right)p_M g\left(e|M\right)h\left(\varepsilon|M\right)V_M + p_H g\left(e|H\right)h\left(\varepsilon|H\right)V_H\right]}{\left(1-\rho\right)\left[p_L g\left(e|L\right)h\left(\varepsilon|L\right)+p_M g\left(e|M\right)h\left(\varepsilon|M\right)\right]+p_H g\left(e|H\right)h\left(\varepsilon|H\right)} de d\varepsilon \\
&> \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{g\left(e|M\right)h\left(\varepsilon|M\right)\left[\left(1-\rho\right)p_M g\left(e|M\right)h\left(\varepsilon|M\right) + p_H g\left(e|H\right)h\left(\varepsilon|H\right)\right]V_M}{\left(1-\rho\right)\left[p_L g\left(e|L\right)h\left(\varepsilon|L\right)+p_M g\left(e|M\right)h\left(\varepsilon|M\right)\right]+p_H g\left(e|H\right)h\left(\varepsilon|H\right)} de d\varepsilon \\
&> \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{g\left(e|M\right)h\left(\varepsilon|M\right)\left(1-\rho\right)p_M g\left(e|M\right)h\left(\varepsilon|M\right)V_M}{\left(1-\rho\right)\left[p_L g\left(e|L\right)h\left(\varepsilon|L\right)+p_M g\left(e|M\right)h\left(\varepsilon|M\right)\right]} de d\varepsilon \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{g\left(e|M\right)h\left(\varepsilon|M\right)p_M g\left(e|M\right)h\left(\varepsilon|M\right)V_M}{\left[p_L g\left(e|L\right)h\left(\varepsilon|L\right)+p_M g\left(e|M\right)h\left(\varepsilon|M\right)\right]} de d\varepsilon \\
&> \int_{-\infty}^{\infty} \frac{g\left(e|M\right)p_M g\left(e|M\right)V_M}{\left[p_L g\left(e|L\right)+p_M g\left(e|M\right)\right]} de = E\left[P\left(e,\emptyset\right)|M\right],
\end{aligned}
$$

where the first inequality comes from $V_H > V_M$; the second inequality holds when $\rho < 1$; and the last inequality,

$$
\int_{-\infty}^{\infty} \frac{g\left(e|M\right)h\left(\varepsilon|M\right)p_M g\left(e|M\right)h\left(\varepsilon|M\right)V_M}{\left[p_L g\left(e|L\right)h\left(\varepsilon|L\right)+p_M g\left(e|M\right)h\left(\varepsilon|M\right)\right]} d\varepsilon > \frac{g\left(e|M\right)p_M g\left(e|M\right)V_M}{\left[p_L g\left(e|L\right)+p_M g\left(e|M\right)\right]},
$$

can be shown using the monotone likelihood ratio property. Therefore, M-managers have incentives to deviate from non-forecasting to forecasting. Thus, it is impossible that only H-managers with discretion choose to forecast in equilibrium, i.e., $\Theta_F^* \neq \{H\}$.

A.3 Proof of Theorem 2

In the proof of Theorem 1, we show that in any equilibrium, that $\forall \rho \in (0, 1]$, we have $\Theta_F^* \notin \left\{\{L\}, \{L, M\}, \{M, H\}, \{L, H\}\right\}$.

Then we only need to show that if $\rho = 1$ (i.e., all managers have discretion) and the support of forecast errors is the same across manager types, $\Theta_F^* \notin \left\{\{M\}, \{H\}\right\}$ in any equilibrium.

(i) $\Theta_F^* \neq \{M\}$: Since only M-managers choose to forecast, the equilibrium market price of a firm with an earnings forecast is:

$$P\left(e,\varepsilon\right) = V_M.$$

On the other hand, since both L and H-managers choose non-forecast, the equilibrium market price of a firm without an earnings forecast is:

$$
P\left(e,\emptyset\right) = \frac{p_H g\left(e|H\right)V_H}{p_L g\left(e|L\right)+p_H g\left(e|H\right)} = \frac{p_H V_H}{p_L \frac{g\left(e|L\right)}{g\left(e|H\right)} + p_H},
$$

which is increasing in e since $\frac{g\left(e|L\right)}{g\left(e|H\right)}$ is decreasing in e. As it is an equilibrium, L-managers do not have an incentive to deviate, i.e.,

$$\begin{aligned}&\Delta\left(L|\Theta_F=\{L,H\}\right)\leq 0\\&\Leftrightarrow E\left[P\left(e,\emptyset\right)|L\right]\geq E\left[P\left(e,\varepsilon\right)|L\right]=V_M,\end{aligned}$$

where $E\left[P\left(e,\varepsilon\right)|L\right]=V_M$ because all managers have the same forecast error support and only M-managers choose to forecast in equilibrium. Moreover, since M-managers have higher expected earnings than L-managers, M-managers have a higher chance to successfully pool with H-managers than L-managers do by not forecasting, i.e.,

$$E\left[P\left(e,\emptyset\right)|M\right]>E\left[P\left(e,\emptyset\right)|L\right],$$

which comes from $P\left(e,\emptyset\right)$ and $\frac{g(e|M)}{g(e|L)}$ being increasing in e. Therefore, we have

$$E\left[P\left(e,\emptyset\right)|M\right]>V_M=E\left[p\left(e,\varepsilon\right)|M\right].$$

This implies that, by withholding earnings forecast, M-managers can increase the price of their firm. Therefore, M-managers have an incentive to deviate from forecasting to non-forecasting. Thus, it is impossible that only M-managers choose to forecast in an equilibrium, i.e., $\Theta_F^*\neq\{M\}$.

(ii) $\Theta_F^*\neq\{H\}$: Since only H-managers choose to forecast, the equilibrium market price of a firm with an earnings forecast is:

$$P\left(e,\varepsilon\right)=V_H.$$

On the other hand, since both L and M-managers choose non-forecast, the equilibrium market price of a firm without an earnings forecast is weakly lower than V_M, i.e., $P\left(e,\emptyset\right)\leq V_M<V_H$.

Because all managers have the same forecast error support and only H-managers choose to forecast in equilibrium, we have $E\left[p\left(e,\varepsilon\right)|L\right]=E\left[P\left(e,\varepsilon\right)|M\right]=V_H$. This implies that by switching to forecasting, L and M-managers can increase the price of their firm. Therefore, L and M-managers have incentives to deviate from non-forecasting to forecasting. Thus, it is impossible that only H-managers choose to forecast in equilibrium, i.e., $\Theta_F^*\neq\{H\}$.

A.4 Proof of Lemma 2

Under the market beliefs in a counter-signaling equilibrium, if a manager does not forecast, the manager's ex-ante expectation of being perceived as an H-manager increases with the manager's true type, i.e., $\zeta\left(L\right)<\zeta\left(M\right)<\zeta\left(H\right)$.

The ex-ante expectation of a θ-manager of being perceived as an H-manager is:

$$\zeta\left(\theta\right)=\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)}{\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1}de.$$

Since the family of densities $g\left(e|\theta\right)$ satisfies the monotone likelihood ratio property, $\frac{g(e|L)}{g(e|H)}$ is decreasing in e and, thus, $\frac{1}{\frac{p_L g(e|L)}{p_H g(e|H)}+1}$ is increasing in e. Using the monotone likelihood ratio property of $g\left(e|\theta\right)$, we obtain $\zeta\left(L\right)<\zeta\left(M\right)<\zeta\left(H\right)$.

A.5 Proof of Proposition 1

We first prove the following two lemmas, which (1) characterize the *valid region* of the ratio V_M/V_H under which counter-signaling arises in equilibrium, and (2) provide the limiting behavior on $\varphi_H\left(\theta\right)$ and $\varphi_M\left(\theta\right)$ as the manager approaches full discretion and as the low type's error becomes infinitely large correspondingly.

Lemma 5. *There exists a partial separating equilibrium in which, among the managers with discretion, M-managers issue guidance, while L and H-managers withhold guidance, if and only if*

$$\frac{\zeta\left(M\right)-\varphi_H\left(M\right)}{\varphi_M\left(M\right)}\leq\frac{V_M}{V_H}\leq\min\left\{\frac{\zeta\left(H\right)-\varphi_H\left(H\right)}{\varphi_M\left(H\right)},\frac{\zeta\left(L\right)-\varphi_H\left(L\right)}{\varphi_M\left(L\right)}\right\}. \tag{15}$$

Proof. First, the condition that type M prefers to forecast is:

$$\begin{aligned}&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g\left(e|M\right)h\left(\varepsilon|M\right)E\left[v|e,\varepsilon\right]ded\varepsilon\geq\int_{-\infty}^{\infty}g\left(e|M\right)E\left[v|e,\emptyset\right]de\\ \Leftrightarrow&\varphi_M\left(M\right)V_M+\varphi_H\left(M\right)V_H\geq\zeta\left(M\right)V_H\\ \Leftrightarrow&\frac{V_M}{V_H}\geq\frac{\zeta\left(M\right)-\varphi_H\left(M\right)}{\varphi_M\left(M\right)}.\end{aligned}$$

Second, the condition that type H prefers not to forecast is:

$$\begin{aligned}&\int_{-\infty}^{\infty}g\left(e|H\right)E\left[v|e,\emptyset\right]de\geq\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g\left(e|H\right)h\left(\varepsilon|H\right)E\left[v|e,\varepsilon\right]ded\varepsilon\\ \Leftrightarrow&\varphi_M\left(H\right)V_M+\varphi_H\left(H\right)V_H\leq\zeta\left(H\right)V_H\\ \Leftrightarrow&\frac{V_M}{V_H}\leq\frac{\zeta\left(H\right)-\varphi_H\left(H\right)}{\varphi_M\left(H\right)}.\end{aligned}$$

Third, the condition that type L prefers not to forecast is:

$$\begin{aligned}&\int_{-\infty}^{\infty}g\left(e|L\right)E\left[v|e,\emptyset\right]de\geq\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g\left(e|L\right)h\left(\varepsilon|L\right)E\left[v|e,\varepsilon\right]ded\varepsilon\\ \Leftrightarrow&\varphi_M\left(L\right)V_M+\varphi_H\left(L\right)V_H\leq\zeta\left(L\right)V_H\\ \Leftrightarrow&\frac{V_M}{V_H}\leq\frac{\zeta\left(L\right)-\varphi_H\left(L\right)}{\varphi_M\left(L\right)}.\end{aligned}$$

Overall, the conditions can be summarized as:

$$\frac{\zeta\left(M\right)-\varphi_H\left(M\right)}{\varphi_M\left(M\right)}\leq\frac{V_M}{V_H}\leq\min\left\{\frac{\zeta\left(H\right)-\varphi_H\left(H\right)}{\varphi_M\left(H\right)},\frac{\zeta\left(L\right)-\varphi_H\left(L\right)}{\varphi_M\left(L\right)}\right\}.$$

□

Lemma 6. *By the definition of $\varphi_H\left(\theta\right)$ and $\varphi_M\left(\theta\right)$, we have $\lim_{\rho\to1}\varphi_M\left(\theta\right)=1$, $\lim_{\rho\to1}\varphi_H\left(\theta\right)=0$, and $\forall\rho<1$, $\lim_{\sigma_L\to\infty}\varphi_M\left(L\right)=\lim_{\sigma_L\to\infty}\varphi_H\left(L\right)=0$.*

Proof. Substituting $h\left(\varepsilon|\theta\right)=\frac{1}{\sigma_\theta\sqrt{2\pi}}\exp\left(-\frac{\varepsilon^2}{2\sigma_\theta^2}\right)$ into $\varphi_M\left(\theta\right)$, we can rewrite $\varphi_M\left(\theta\right)$ as:

$$
\begin{aligned}
&\varphi_M\left(\theta\right)\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)h\left(\varepsilon|\theta\right)}{\left(1-\rho\right)\left[\frac{p_L}{p_M}\frac{g(e|L)h(\varepsilon|L)}{g(e|M)h(\varepsilon|M)}+\frac{p_H}{p_M}\frac{g(e|H)h(\varepsilon|H)}{g(e|M)h(\varepsilon|M)}\right]+1}ded\varepsilon\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)\frac{1}{\sigma_\theta\sqrt{2\pi}}\exp\left(-\frac{\varepsilon^2}{2\sigma_\theta^2}\right)}{\left(1-\rho\right)\left[\frac{p_L}{p_M}\frac{g(e|L)}{g(e|M)}\frac{\sigma_M}{\sigma_L}\exp\left(\frac{\varepsilon^2}{2\sigma_M^2}-\frac{\varepsilon^2}{2\sigma_L^2}\right)+\frac{p_H}{p_M}\frac{g(e|H)}{g(e|M)}\frac{\sigma_M}{\sigma_H}\exp\left(\frac{\varepsilon^2}{2\sigma_M^2}-\frac{\varepsilon^2}{2\sigma_H^2}\right)\right]+1}ded\varepsilon.
\end{aligned}
$$

Let $\epsilon=\frac{\varepsilon}{\sigma_\theta}$, we can get

$$
\begin{aligned}
&\varphi_M\left(\theta\right)\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{r^2}{2}\right)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\epsilon^2}{2}\right)}{\left(1-\rho\right)\left[\frac{p_L}{p_M}\frac{g(e|L)}{g(e|M)}\frac{\sigma_M}{\sigma_L}\exp\left[\frac{\sigma_\theta^2}{2}\left(\frac{\epsilon^2}{\sigma_M^2}-\frac{\epsilon^2}{\sigma_L^2}\right)\right]+\frac{p_H}{p_M}\frac{g(e|H)}{g(e|M)}\frac{\sigma_M}{\sigma_H}\exp\left[\frac{\sigma_\theta^2}{2}\left(\frac{\epsilon^2}{\sigma_M^2}-\frac{\epsilon^2}{\sigma_H^2}\right)\right]\right]+1}drd\epsilon.
\end{aligned}
$$

Therefore, when $\rho<1$, we have

$$\lim_{\sigma_L\to\infty}\varphi_M\left(L\right)=0,\ \text{and}\ \lim_{\rho\to 1}\varphi_M\left(\theta\right)=1.$$

Similarly, substituting $h\left(\varepsilon|\theta\right)=\frac{1}{\sigma_\theta\sqrt{2\pi}}\exp\left(-\frac{\varepsilon^2}{2\sigma_\theta^2}\right)$ into $\varphi_H\left(\theta\right)$, we obtain

$$
\begin{aligned}
\varphi_H\left(\theta\right)&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)h\left(\varepsilon|\theta\right)}{\left[\frac{p_L}{p_H}\frac{g(e|L)h(\varepsilon|L)}{g(e|H)h(\varepsilon|H)}+1\right]+\frac{1}{(1-\rho)}\frac{p_M}{p_H}\frac{g(e|M)h(\varepsilon|M)}{g(e|H)h(\varepsilon|H)}}ded\varepsilon\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)\frac{1}{\sigma_\theta\sqrt{2\pi}}\exp\left(-\frac{\varepsilon^2}{2\sigma_\theta^2}\right)}{\left[\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}\frac{\sigma_H}{\sigma_L}\exp\left(\frac{\varepsilon^2}{2\sigma_H^2}-\frac{\varepsilon^2}{2\sigma_L^2}\right)+1\right]+\frac{1}{(1-\rho)}\frac{p_M}{p_H}\frac{g(e|M)}{g(e|H)}\frac{\sigma_H}{\sigma_M}\exp\left(\frac{\varepsilon^2}{2\sigma_H^2}-\frac{\varepsilon^2}{2\sigma_M^2}\right)}ded\varepsilon.
\end{aligned}
$$

Let $\epsilon=\frac{\varepsilon}{\sigma_\theta}$, we can get

$$
\begin{aligned}
&\varphi_H\left(\theta\right)\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{g\left(e|\theta\right)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\epsilon^2}{2}\right)}{\left[\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}\frac{\sigma_H}{\sigma_L}\exp\left(\frac{\sigma_\theta^2}{2}\left(\frac{\epsilon^2}{\sigma_H^2}-\frac{\epsilon^2}{\sigma_L^2}\right)\right)+1\right]+\frac{1}{(1-\rho)}\frac{p_M}{p_H}\frac{g(e|M)}{g(e|H)}\frac{\sigma_H}{\sigma_M}\exp\left(\frac{\sigma_\theta^2}{2}\left(\frac{\epsilon^2}{\sigma_H^2}-\frac{\epsilon^2}{\sigma_M^2}\right)\right)}ded\epsilon.
\end{aligned}
$$

Therefore, when $\rho < 1$, we have

$$\lim_{\sigma_L \to \infty} \varphi_H(L) = 0, \text{ and } \lim_{\rho \to 1} \varphi_H(\theta) = 0.$$

□

The result in Proposition 1 follows immediately from Lemmas 2, 5 and 6.

A.6 Proof of Lemma 3

In an equilibrium in which only type M chooses to forecast, while both type L and H choose non-forecast, the equilibrium price satisfies

$$\begin{aligned} P\left(e, \emptyset\right) = E\left[v | e, \emptyset\right] &= \frac{p_L g\left(e|L\right) V_L + p_H g\left(e|H\right) V_H}{p_L g\left(e|L\right) + p_H g\left(e|H\right)} \\ &= \frac{p_H g\left(e|H\right) V_H}{p_L g\left(e|L\right) + p_H g\left(e|H\right)} = \frac{V_H}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)} + 1}, \end{aligned}$$

and

$$P\left(e, \varepsilon\right) = \begin{cases} 0 & \text{if } |y - e| > b_M; \\ V_M & \text{if } |y - e| \leq b_M. \end{cases}$$

First, the condition that type M prefers to forecast is:

$$\begin{aligned} &\int_{-\infty}^{\infty} \int_{-b_M}^{b_M} g\left(e|M\right) h\left(\varepsilon|M\right) P\left(e, \varepsilon\right) d\varepsilon de \geq \int_{-\infty}^{\infty} g\left(e|M\right) P\left(e, \emptyset\right) de \\ \Leftrightarrow & V_M \geq \int_{-\infty}^{\infty} g\left(e|M\right) \frac{V_H}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)} + 1} de \\ \Leftrightarrow & \frac{V_M}{V_H} \geq \int_{-\infty}^{\infty} g\left(e|M\right) \frac{1}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)} + 1} de. \end{aligned}$$

Second, the conditions that types L and H prefer not to forecast are:

(i) type H:

$$\begin{aligned} &\int_{-\infty}^{\infty} g\left(e|H\right) P\left(e, \emptyset\right) de \geq \int_{-\infty}^{\infty} \int_{-b_H}^{b_H} g\left(e|H\right) h\left(\varepsilon|H\right) P\left(e, \varepsilon\right) d\varepsilon de \\ \Leftrightarrow & \int_{-\infty}^{\infty} g\left(e|H\right) \frac{V_H}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)} + 1} de \geq V_M \\ \Leftrightarrow & \frac{V_M}{V_H} \leq \int_{-\infty}^{\infty} g\left(e|H\right) \frac{1}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)} + 1} de; \end{aligned}$$

(ii) type L:

$$\int_{-\infty}^{\infty} g\left(e|L\right) P\left(e,\emptyset\right) de \geq \int_{-\infty}^{\infty}\int_{-b_L}^{b_L} g\left(e|L\right) h\left(\varepsilon|L\right) P\left(e,\varepsilon\right) d\varepsilon de$$

$$\Leftrightarrow \int_{-\infty}^{\infty} g\left(e|L\right) \frac{V_H}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)}+1} de \geq \frac{b_M}{b_L} V_M$$

$$\Leftrightarrow \frac{V_M}{V_H} \leq \frac{b_L}{b_M}\int_{-\infty}^{\infty} g\left(e|L\right) \frac{1}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)}+1} de.$$

Overall, the conditions are summarized as:

$$\zeta\left(M\right) \leq \frac{V_M}{V_H} \leq \min\left\{\frac{b_L}{b_M}\zeta\left(L\right), \zeta\left(H\right)\right\}.$$

A.7 Proof of Lemma 4

In an equilibrium where $\Theta_F^* = \{H\}$, the market price for a firm without an earning forecast is determined by:

$$P\left(e,\emptyset\right) = E\left[v|e,\emptyset\right] = \frac{p_M g\left(e|M\right) V_M + p_H g\left(e|H\right) V_L}{p_M g\left(e|M\right) + p_L g\left(e|L\right)} = \frac{V_M}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1}.$$

The market price of a firm with a forecast $y = e + \varepsilon$ and an earnings report e is:

$$P\left(e,\varepsilon\right) = \begin{cases} 0 & \text{if } |\varepsilon| > b_H; \\ V_H & \text{if } |\varepsilon| \leq b_H. \end{cases}$$

The conditions that L and M-managers prefer not to forecast are:

$$\frac{b_H}{b_M} V_H \leq \int_{-\infty}^{\infty} \frac{g\left(e|M\right) V_M}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} de, \text{ and } \frac{b_H}{b_L} V_H \leq \int_{-\infty}^{\infty} \frac{g\left(e|L\right) V_M}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} de.$$

The condition that H-managers prefer to forecast:

$$V_H \geq \int_{-\infty}^{\infty} \frac{g\left(e|H\right) V_M}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} de,$$

which holds as long as $V_H \geq V_M$, which is true by assumption.

Overall, the conditions can be summarized as:

$$\frac{V_H}{V_M} \leq \min\left\{\frac{b_M}{b_H}\int_{-\infty}^{\infty} \frac{g\left(e|M\right)}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} de, \frac{b_L}{b_H}\int_{-\infty}^{\infty} \frac{g\left(e|L\right)}{\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} de\right\}.$$

A.8 Proof of Proposition 2

The result follows from the conditions for the existence of equilibrium where $\Theta_F^* = \{H\}$ (see Lemma 3) and $\Theta_F^* = \{M\}$ (see Lemma 4).

A.9 Proof of Proposition 3

In an equilibrium, M-managers with discretion and managers without discretion choose to forecast, while both type L and H choose to remain quiet. The equilibrium price satisfies

$$\begin{aligned}
P(e,\varepsilon) &= \frac{p_M \frac{1_{|\varepsilon|\le b_M}}{b_M} g\left(e|M\right) V_M + (1-\rho)\, p_H \frac{1_{|\varepsilon|\le b_H}}{b_H} g\left(e|H\right) V_H}{(1-\rho)\left[p_L \frac{1_{|\varepsilon|\le b_L}}{b_L} g\left(e|L\right) + p_H \frac{1_{|\varepsilon|\le b_H}}{b_H} g\left(e|H\right)\right] + p_M \frac{1_{|\varepsilon|\le b_M}}{b_M} g\left(e|M\right)} \\
&= \begin{cases} 0 & \text{if } |\varepsilon| > b_M; \\ \frac{V_M}{(1-\rho)\frac{p_L g(e|L)}{p_M g(e|M)}+1} & \text{if } b_H < |\varepsilon| \le b_M; \\ \frac{\frac{p_M g(e|M)}{p_H g(e|H)} V_M + (1-\rho) V_H}{(1-\rho)\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{p_H g(e|H)}} & \text{if } |\varepsilon| \le b_H, \end{cases}
\end{aligned}$$

and

$$P\left(e,\emptyset\right) = E\left[v|e,\emptyset\right] = \frac{V_H}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1}.$$

First, the condition that type M prefers forecast to non forecast is:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-b_M}^{b_M} g\left(e|M\right) h\left(\varepsilon|M\right) P\left(e,\varepsilon\right) d\varepsilon de \ge \int_{-\infty}^{\infty} g\left(e|M\right) P\left(e,\emptyset\right) de \\
\Leftrightarrow &\int_{-\infty}^{\infty} g\left(e|M\right)\left\{\frac{b_M-b_H}{b_M}\frac{V_M}{(1-\rho)\frac{p_L g(e|L)}{p_M g(e|M)}+1} + \frac{b_H}{b_M}\frac{\frac{p_M g(e|M)}{p_H g(e|H)} V_M + (1-\rho) V_H}{(1-\rho)\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{p_H g(e|H)}}\right\} de \\
&\ge \int_{-\infty}^{\infty} g\left(e|M\right)\frac{V_H}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1} de,
\end{aligned}$$

which can be simplified as:

$$\frac{V_M}{V_H} \ge \frac{\int_{-\infty}^{\infty} g\left(e|M\right)\left\{\frac{1}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1} - \frac{b_H}{b_M}\frac{1}{\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{(1-\rho)p_H g(e|H)}}\right\} de}{\int_{-\infty}^{\infty} g\left(e|M\right)\left\{\frac{b_M-b_H}{b_M}\frac{1}{(1-\rho)\frac{p_L g(e|L)}{p_M g(e|M)}+1} + \frac{b_H}{b_M}\frac{\frac{p_M g(e|M)}{p_H g(e|H)}}{(1-\rho)\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{p_H g(e|H)}}\right\} de}.$$

Second, the conditions that type L and H prefers non-forecast to forecast are:

(i) type H:

$$\int_{-\infty}^{\infty} g\left(e|H\right) P\left(e,\emptyset\right) de \geq \int_{-\infty}^{\infty}\int_{-b_H}^{b_H} g\left(e|H\right) h\left(\varepsilon|H\right) P\left(e,\varepsilon\right) d\varepsilon de$$

$$\Leftrightarrow \int_{-\infty}^{\infty} g\left(e|H\right) \frac{V_H}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)}+1} de \geq \int_{-\infty}^{\infty} g\left(e|H\right) \left\{ \frac{\frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)} V_M + (1-\rho) V_H}{(1-\rho)\left[\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1\right] + \frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)}} \right\} de$$

$$\Leftrightarrow \frac{V_M}{V_H} \leq \frac{\int_{-\infty}^{\infty} g\left(e|H\right) \left\{ \frac{1}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)}+1} - \frac{1}{\left[\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1\right] + \frac{p_M g\left(e|M\right)}{(1-\rho)p_H g\left(e|H\right)}} \right\} de}{\int_{-\infty}^{\infty} g\left(e|H\right) \left\{ \frac{\frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)}}{(1-\rho)\left[\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1\right] + \frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)}} \right\} de};$$

(ii) type L:

$$\int_{-\infty}^{\infty} g\left(e|L\right) P\left(e,\emptyset\right) de \geq \int_{-\infty}^{\infty}\int_{-b_L}^{b_L} g\left(e|L\right) h\left(\varepsilon|L\right) P\left(e,\varepsilon\right) d\varepsilon de$$

$$\Leftrightarrow \int_{-\infty}^{\infty} g\left(e|L\right) \frac{V_H}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)}+1} de$$

$$\geq \int_{-\infty}^{\infty} g\left(e|L\right) \left\{ \frac{b_M - b_H}{b_L} \frac{V_M}{(1-\rho)\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} + \frac{b_H}{b_L} \frac{\frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)} V_M + (1-\rho) V_H}{(1-\rho)\left[\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1\right] + \frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)}} \right\} de$$

$$\Leftrightarrow \frac{V_M}{V_H} \leq \frac{\int_{-\infty}^{\infty} g\left(e|L\right) \left\{ \frac{1}{\frac{p_L}{p_H}\frac{g\left(e|L\right)}{g\left(e|H\right)}+1} - \frac{b_H}{b_L}\frac{V_H}{\left[\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1\right] + \frac{p_M g\left(e|M\right)}{(1-\rho)p_H g\left(e|H\right)}} \right\} de}{\int_{-\infty}^{\infty} g\left(e|L\right) \left\{ \frac{b_M-b_H}{b_L} \frac{1}{(1-\rho)\frac{p_L g\left(e|L\right)}{p_M g\left(e|M\right)}+1} + \frac{b_H}{b_L} \frac{\frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)}}{(1-\rho)\left[\frac{p_L g\left(e|L\right)}{p_H g\left(e|H\right)}+1\right] + \frac{p_M g\left(e|M\right)}{p_H g\left(e|H\right)}} \right\} de}.$$

Define the following ratios:

$$R_M = \frac{\int_{-\infty}^{\infty} g\left(e|M\right)\left\{\frac{1}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1} - \frac{b_H}{b_M}\frac{1}{\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{(1-\rho)p_H g(e|H)}}\right\}de}{\int_{-\infty}^{\infty} g\left(e|M\right)\left\{\frac{b_M-b_H}{b_M}\frac{1}{(1-\rho)\frac{p_L g(e|L)}{p_M g(e|M)}+1} + \frac{b_H}{b_M}\frac{\frac{p_M g(e|M)}{p_H g(e|H)}}{(1-\rho)\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{p_H g(e|H)}}\right\}de},$$

$$R_H = \frac{\int_{-\infty}^{\infty} g\left(e|H\right)\left\{\frac{1}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1} - \frac{1}{\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{(1-\rho)p_H g(e|H)}}\right\}de}{\int_{-\infty}^{\infty} g\left(e|H\right)\left\{\frac{\frac{p_M g(e|M)}{p_H g(e|H)}}{(1-\rho)\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{p_H g(e|H)}}\right\}de},$$

$$R_L = \frac{\int_{-\infty}^{\infty} g\left(e|L\right)\left\{\frac{1}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1} - \frac{b_H}{b_L}\frac{V_H}{\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{(1-\rho)p_H g(e|H)}}\right\}de}{\int_{-\infty}^{\infty} g\left(e|L\right)\left\{\frac{b_M-b_H}{b_L}\frac{1}{(1-\rho)\frac{p_L g(e|L)}{p_M g(e|M)}+1} + \frac{b_H}{b_L}\frac{\frac{p_M g(e|M)}{p_H g(e|H)}}{(1-\rho)\left[\frac{p_L g(e|L)}{p_H g(e|H)}+1\right]+\frac{p_M g(e|M)}{p_H g(e|H)}}\right\}de}.$$

Overall, the conditions are summarized as: $R_M \leq \frac{V_M}{V_H} \leq \min\left\{R_L, R_H\right\}$.

Note that, $\lim_{b_L\to\infty} R_L = \infty$, and

$$\lim_{b_H\to b_M}\lim_{\rho\to 1}\left(R_H - R_M\right) = \int_{-\infty}^{\infty}\left[g\left(e|H\right) - g\left(e|M\right)\right]\frac{1}{\frac{p_L}{p_H}\frac{g(e|L)}{g(e|H)}+1}de > 0,$$

which is true by the monotone likelihood ratio property. Therefore, by the continuity of the function R_L on b_L and the function R_M, R_H on b_H, ρ, when $\frac{b_L}{b_M}$, $\frac{b_H}{b_M}$ and ρ are large enough, there exist $\underline{r}$ and $\overline{r}$, s.t, $\underline{r} = R_M < \min\left\{R_L, R_H\right\} = \overline{r}$, and a counter-signaling equilibrium exists if and only if $\frac{V_M}{V_H} \in \left(\underline{r}, \overline{r}\right)$.

A.10 Proof of Proposition 4

It is equivalent to prove that $\forall\theta^* \in (\theta_{\min}, \theta_{\max})$, $\Theta_F^* \neq [\theta_{\min}, \theta^*]$ and $\Theta_F^* \neq [\theta^*, \theta_{\max}]$.

First, we show that $\Theta_F^* \neq [\theta^*, \theta_{\max}]$. Suppose there is an equilibrium in which $\Theta_F^* = [\theta^*, \theta_{\max}]$. Then, it implies $\forall\theta \in [\theta^*, \theta_{\max}]$, $\Delta\left(\theta^* | \left[\theta^*, \theta_{\max}\right]\right) \geq 0$, and $\forall\theta \in [\theta_{\min}, \theta^*]$, $\Delta\left(\theta^* | \left[\theta^*, \theta_{\max}\right]\right) \leq 0$. By continuity, we have $\Delta\left(\theta^* | \left[\theta^*, \theta_{\max}\right]\right) = 0$.

Under such an equilibrium, the market price of a firm with earnings report e and earnings forecast error ε is:

$$\begin{aligned}
P(e,\varepsilon) &= \frac{\int_{\theta_{\min}}^{\theta_{\max}} (1-\rho) f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta + \int_{\theta^*}^{\theta_{\max}} \rho f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta_{\max}} (1-\rho) f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta + \int_{\theta^*}^{\theta_{\max}} \rho f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} \\
&= \frac{\int_{\theta_{\min}}^{\theta^*} (1-\rho) f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta + \int_{\theta^*}^{\theta_{\max}} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} (1-\rho) f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta + \int_{\theta^*}^{\theta_{\max}} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} \\
&> \frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta},
\end{aligned}$$

and the market price of a firm with earnings report e only is:

$$P\left(e,\emptyset\right) = \frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) d\theta}.$$

We can show that

$$\begin{aligned}
\frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) d\theta} &\leq \int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) \frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} d\varepsilon \\
&< \int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) p(e,\varepsilon)\, d\varepsilon,
\end{aligned}$$

which implies $\Delta\left(\theta^*|\left[\theta^*,\theta_{\max}\right]\right) > 0$, leading to a contradiction. The proof of the first inequality is as follows:

(i) First, by the monotone likelihood ratio property and $v'(\theta) > 0$, we have $\forall \theta, s \in [\theta_{\min}, \theta^*]$,

$$[v_\theta - v_s] \int_{-\infty}^{\infty} \left[\frac{h\left(\varepsilon|\theta\right) - h\left(\varepsilon|s\right)}{\int_{\theta_{\min}}^{\theta^*} f(\theta') g\left(e|\theta'\right) \frac{h\left(\varepsilon|\theta'\right)}{h\left(\varepsilon|\theta^*\right)} d\theta'} \right] d\varepsilon \geq 0.$$

(ii) Then, we can get

$$\begin{aligned}
&\int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) \frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} d\varepsilon - \frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) d\theta} \\
&= \int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) \left[\frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} - \frac{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta^*} f(\theta) g\left(e|\theta\right) d\theta} \right] d\varepsilon,
\end{aligned}$$

which can be reorganized as:

$$\int_{-\infty}^{\infty}\left[\frac{\int_{\theta_{\min}}^{\theta^*}\int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) f(s)\, g\left(e|s\right)\left[h\left(\varepsilon|\theta\right) v_\theta - h\left(\varepsilon|s\right) v_\theta\right] d\theta ds}{\int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right)\frac{h\left(\varepsilon|\theta\right)}{h\left(\varepsilon|\theta^*\right)} d\theta \int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) d\theta}\right] d\varepsilon$$

$$= \int_{-\infty}^{\infty}\left[\frac{\int_{\theta_{\min}}^{\theta^*}\int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) f(s)\, g\left(e|s\right)\left[h\left(\varepsilon|\theta\right) - h\left(\varepsilon|s\right)\right]\left[v_\theta - v_s\right] d\theta ds}{2\int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) d\theta \int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right)\frac{h\left(\varepsilon|\theta\right)}{h\left(\varepsilon|\theta^*\right)} d\theta}\right] d\varepsilon$$

$$= \frac{\int_{\theta_{\min}}^{\theta^*}\int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) f(s)\, g\left(e|s\right)\left[v_\theta - v_s\right]\int_{-\infty}^{\infty}\left[\frac{h\left(\varepsilon|\theta\right) - h\left(\varepsilon|s\right)}{\int_{\theta_{\min}}^{\theta^*} f(\theta')\, g\left(e|\theta'\right)\frac{h\left(\varepsilon|\theta'\right)}{h\left(\varepsilon|\theta^*\right)} d\theta'}\right] d\varepsilon d\theta ds}{2\int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) d\theta}$$

$$\geq 0.$$

Second, we show that $\Theta_F^* \neq [\theta_{\min}, \theta^*]$. Suppose there is an equilibrium in which $\Theta_F^* = [\theta_{\min}, \theta^*]$. Then, it implies $\forall\theta \in [\theta^*, \theta_{\max}]$, $\Delta\left(\theta^* | [\theta^*, \theta_{\max}]\right) \leq 0$, and $\forall\theta \in [\theta_{\min}, \theta^*]$, $\Delta\left(\theta^* | [\theta^*, \theta_{\max}]\right) \geq 0$. By continuity, we have $\Delta\left(\theta^* | [\theta_{\min}, \theta^*]\right) = 0$.

Under such an equilibrium, the market price of a firm with earnings report e and earnings forecast error ε is:

$$\begin{aligned}
P(e,\varepsilon) &= \frac{\int_{\theta_{\min}}^{\theta_{\max}} (1-\rho) f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta + \int_{\theta_{\min}}^{\theta^*} \rho f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta_{\min}}^{\theta_{\max}} (1-\rho) f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta + \int_{\theta_{\min}}^{\theta^*} \rho f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} \\
&= \frac{\int_{\theta^*}^{\theta_{\max}} (1-\rho) f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta + \int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} (1-\rho) f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta + \int_{\theta_{\min}}^{\theta^*} f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} \\
&< \frac{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta},
\end{aligned}$$

and the market price of a firm with earnings report e only is:

$$P\left(e,\emptyset\right) = \frac{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) d\theta}.$$

We can show that

$$\begin{aligned}
\frac{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) d\theta} &\geq \int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right)\frac{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f(\theta)\, g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} d\varepsilon \\
&> \int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) p(e,\varepsilon)\, d\varepsilon,
\end{aligned}$$

which implies $\Delta\left(\theta^* | [\theta^*, \theta_{\max}]\right) < 0$, leading to a contradiction. The proof of the first inequality is as follows:

(i) First, by the monotone likelihood ratio property and $v'(\theta) > 0$, we have $\forall \theta, s \in [\theta^*, \theta_{\max}]$,

$$[v_\theta - v_s] \int_{-\infty}^{\infty} \left[\frac{h\left(\varepsilon|\theta\right) - h\left(\varepsilon|s\right)}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta'\right) g\left(e|\theta'\right) \frac{h\left(\varepsilon|\theta'\right)}{h\left(\varepsilon|\theta^*\right)} d\theta'} \right] d\varepsilon \leq 0.$$

(ii) Then, we can get

$$\begin{aligned}
&\int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) \frac{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} d\varepsilon - \frac{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) d\theta} \\
&= \int_{-\infty}^{\infty} h\left(\varepsilon|\theta^*\right) \left[\frac{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) h\left(\varepsilon|\theta\right) d\theta} - \frac{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) v_\theta d\theta}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) d\theta} \right] d\varepsilon
\end{aligned}$$

which can be reorganized as:

$$\begin{aligned}
&\int_{-\infty}^{\infty} \left[\frac{\int_{\theta^*}^{\theta_{\max}} \int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) f\left(s\right) g\left(e|s\right) \left[h\left(\varepsilon|\theta\right) v_\theta - h\left(\varepsilon|s\right) v_\theta\right] d\theta ds}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) \frac{h\left(\varepsilon|\theta\right)}{h\left(\varepsilon|\theta^*\right)} d\theta \int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) d\theta} \right] d\varepsilon \\
&= \int_{-\infty}^{\infty} \left[\frac{\int_{\theta^*}^{\theta_{\max}} \int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) f\left(s\right) g\left(e|s\right) \left[h\left(\varepsilon|\theta\right) - h\left(\varepsilon|s\right)\right] [v_\theta - v_s] d\theta ds}{2 \int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) d\theta \int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) \frac{h\left(\varepsilon|\theta\right)}{h\left(\varepsilon|\theta^*\right)} d\theta} \right] d\varepsilon \\
&= \frac{\int_{\theta^*}^{\theta_{\max}} \int_{\theta^*}^{\theta_{\max}} f\left(\theta\right) g\left(e|\theta\right) f\left(s\right) g\left(e|s\right) [v_\theta - v_s] \int_{-\infty}^{\infty} \left[\frac{h\left(\varepsilon|\theta\right) - h\left(\varepsilon|s\right)}{\int_{\theta^*}^{\theta_{\max}} f\left(\theta'\right) g\left(e|\theta'\right) \frac{h\left(\varepsilon|\theta'\right)}{h\left(\varepsilon|\theta^*\right)} d\theta'} \right] d\varepsilon d\theta ds}{2 \int_{\theta_{\min}}^{\theta^*} f\left(\theta\right) g\left(e|\theta\right) d\theta} \\
&\leq 0.
\end{aligned}$$